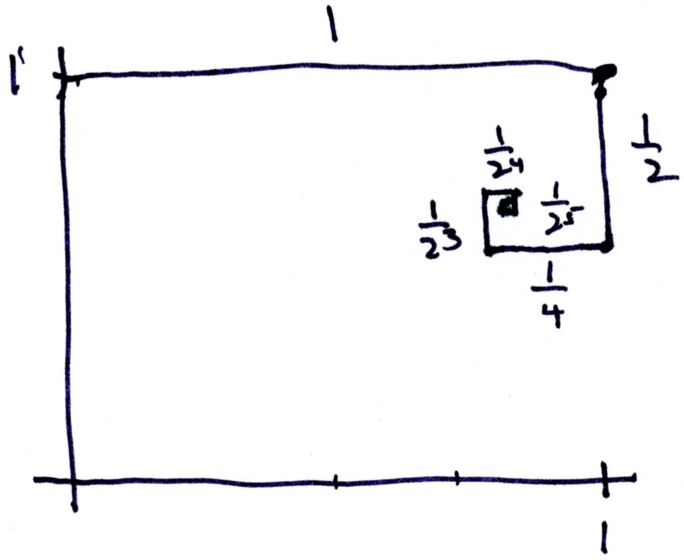
ZENO'S PARADOX

$$a_1 + a_2 + \cdots = \sum_{i=1}^{\infty} a_i = \infty$$

$$\sum_{i=1}^{\infty} \frac{1}{2^i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \infty$$

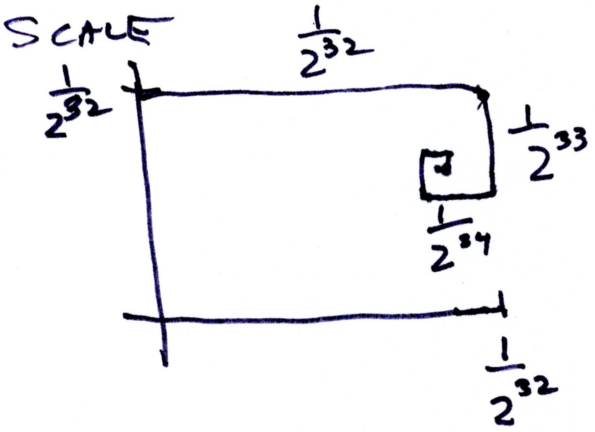
GEOMETRIC VISUALIZATION

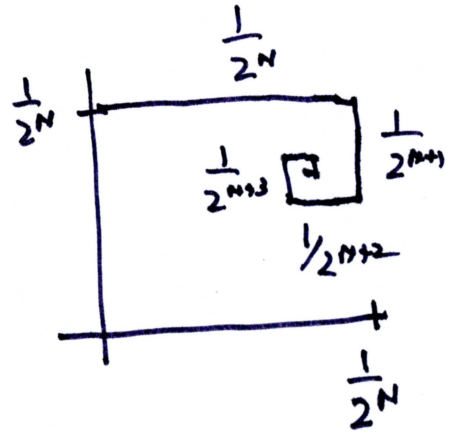


(a, b) LIMIT POINT

$(\frac{4}{5}, \frac{3}{5})$

SCALE





GEOMETRIC SERIES

$$a + ar + ar^2 + \cdots = \sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}$$

$|r| < 1$

$$\sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1-\frac{1}{2}} = 2$$

AFTER 100 SEGMENTS

$$1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{100}} = 2 - \frac{1}{2^{100}}$$

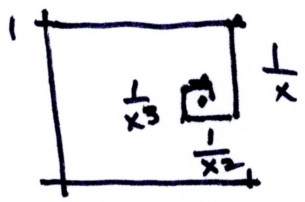
EXERCISE

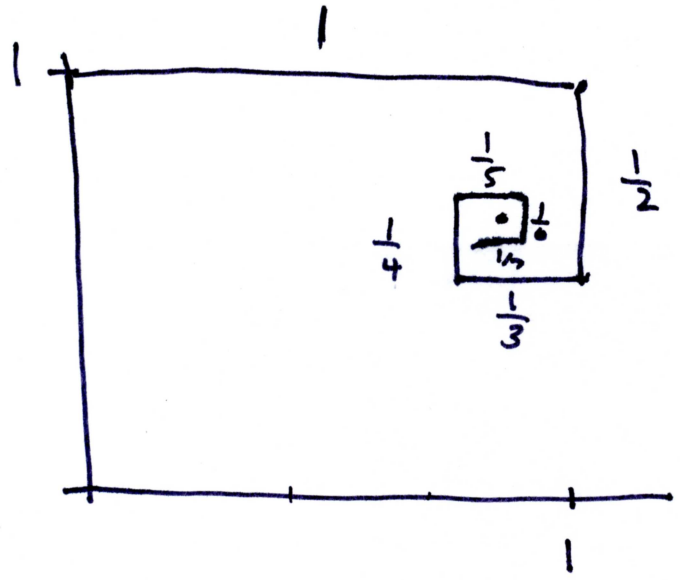
1) PROVE $(a,b) = \left(\frac{4}{5}, \frac{3}{5}\right)$

2) $x > 1$ $\quad \sum_{i=0}^{\infty} \frac{1}{x^i} = \frac{x}{x-1}$

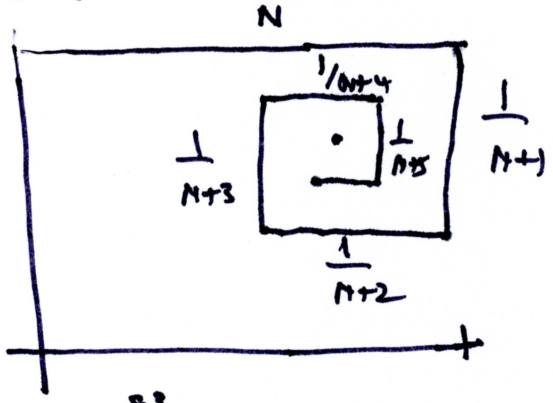
3) $x > 1$ $\quad \sum_{i=0}^{\infty} (-1)^i \cdot \frac{1}{x^i} = \frac{x}{x+1}$

4) FOR $x > 1$ $\quad (a,b) = \left(\frac{x^2}{x^2+1}, \frac{x^2+1-x}{x^2+1}\right)$





SCALE $\frac{1}{N}$



$$\sum_{i=1}^{83} \frac{1}{i} > 5$$

$$\sum_{i=1}^{12,367} \frac{1}{i} > 10$$

$$\sum_{i=1}^{250,000,000} \frac{1}{i} > 20$$

SIZE OF ATOM

LIMIT?

TOTAL LENGTH?

UPPER BOUND? NONE

TOTAL LENGTH ∞

1350 NICOLE ORESME

$$1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{>\frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{>\frac{1}{2}} + \underbrace{\frac{1}{9} + \cdots + \frac{1}{16}}_{>\frac{1}{2}}$$

$$\underbrace{\frac{1}{2^m+1} + \frac{1}{2^m+2} + \cdots + \frac{1}{2^m+2^m}}_{2^m \text{ TERMS}}$$

$$> \frac{1}{2^{m+1}} + \frac{1}{2^{m+1}} + \cdots \frac{1}{2^{m+1}} = 2^m \cdot \frac{1}{2^{m+1}} = \frac{1}{2}$$

EXERCISE: FIND (a, b)

ANSWER: $\left(\frac{\pi}{4}, 1 + \ln\left(\frac{\sqrt{2}}{2}\right) \right)$

WOLFRAM ALPHA

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots = \frac{\pi}{4}$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = LN(2) = \log_e(2)$$

PROVE: $a = \frac{\pi}{4}$

$$b = 1 - \frac{1}{2} LN(2)$$

$$= 1 + LN\, 2^{(-1/2)}$$

$$= 1 + LN\, \frac{1}{\sqrt{2}}$$

$$= 1 + LN\left(\frac{\sqrt{2}}{2}\right)$$

ANSWER $(a, b) = \left(\frac{\pi}{4},\ 1 + LN\left(\frac{\sqrt{2}}{2}\right)\right)$

FRACTAL GEOMETRY

1) SCALING

2) BOUNDARIES WITH ∞ LENGTH (AREA) AROUND FINITE AREAS

3.) FRACTIONAL DIMENSIONS

4) NON-SMOOTH

5) GEOMETRY OF NATURE

EUCLIDEAN + (NON-EUCLIDEAN) GEOMETRIES

SMOOTH

INTEGER DIMENSIONS

MAN MADE TECHNOLOGIES.