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21st Century Mathematics

Teaching Math

A High School Math Curriculum

That Works for All Students

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You may learn more about him at:

www.TriadMathInc.com

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By: Craig Hane, Ph.D.

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Dedicated with love, respect, and thanks to:

Miss Madonna O'Hair

Dr. Clint Gass

Dr. Robert Stoll

Dr. George Whaples

Mrs. Betty Unsworth

Fred Unsworth

My family

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FORWARD Author's Comments

This is a book for anyone interested in high school mathematics and its presentation to students.

It is particularly aimed at anyone who has an interest in a student who is struggling with math or is simply not satisfied with a student's high school math education program.

While quite critical of the existing "standard math curriculum" taught at most schools I offer one potential solution that I know, from over fifty years of teaching math at all post elementary levels, including adult technical training, will work for any student.

In the course of this book I will direct you to many resources, some free, and some not free, which may help you. Today there really is no valid excuse with the modern technologies and tools available cost effectively today to have any failing math students.

The Effective Math Education TRIAD, Chapter 2, is the underlying set of principles that guide everything I will recommend to you.

I wish you well in your future endeavors in learning or teaching math at the post elementary level whether your student is going to pursue a non-professional technical career or job, or a demanding STEM profession, or even become a professional math-ematician.

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Preface - - - A Story

The fourteen year old boy was very discouraged. A **B-** in freshman Algebra was his final grade for the year. What had happened?

He was always good at math until this year. He liked arithmetic calculations and was pretty good at them. In fact, better than many other students in contests. So why was he "no good" at algebra?

His teacher, Mr. H., was a laid-back no nonsense humorless type of guy. The boy couldn't seem to communicate with him very well. He had pretty much given up asking questions since they were usually not answered in a way he could understand.

The boy decided he just didn't like math if algebra was what math consisted of. Next year he had another teacher, Miss O. for another type of math, Geometry, whatever that was. He kind of dreaded it. If he wasn't any good at algebra, why would geometry be any better? Turned out, he was dead wrong.

Geometry was all about real physical types of things. He found it to be a lot of fun and easy to understand. Algebra had been about a bunch of X's and algorithms and "word problems" that seemed contrived and artificial. What would one ever do with the quadratic formula? Indeed, what did it mean?

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In geometry, he learned about triangles and circles and polygons and how to solve problems using his intuition and reasoning powers. He once again found the pleasure in math he had enjoyed earlier in arithmetic. He made an **A** once again.

Then, in his junior year he had Mr. H again, this time for Solid Geometry. And, once again he didn't enjoy math. Only this time, he had something to compare to. He decided it wasn't so much the subject, but how it was being taught to him.

He lamented his situation to Miss O. She didn't say anything negative about Mr. H. But, she did recommend to the boy that the next year, his last year in high school, he take math courses at the local college as a special student.

Over the objections of his principal and advisor, and with fear and trepidation, he enrolled in a College Algebra course taught by Professor G.

To his amazement he now found algebra was as enjoyable as geometry, and he earned an **A** again. He was overjoyed. How math was taught mattered!

He went on to another college and majored in math. He found that many times how much he enjoyed and understood a new math subject was very much a function of the teacher.

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He began to tutor his fellow students, and found most of them had had similar experiences and really liked and understood math when it was taught in a proper way. In fact, he earned his living as a math tutor of his fellow students and enjoyed it so much he just wanted to continue his math education and teaching.

Right after earning his B.A. he taught high school math for a year. He loved it and found most of his students could learn math IF they were given enough attention. However, that was very difficult to do in the large classes he had. It was much different than tutoring his classmates.

If only he had the time to tutor them all! They deserved it he felt. But, it was simply logistically and financially impossible. So, he went back to studying math in graduate school and tutoring other students.

He found any student could learn math if tutored properly if the student would put in the effort. He also learned that most students learned to enjoy math once they overcame their fears like he once had.

Eventually he was given a Ph. D. and "kicked out" of his life as a student. So ended the boy's "childhood".

He went on to be a math professor for several years. It was better than teaching high school, because his college students could select their courses and only the best motivated students took his courses. Thus, he was

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very successful teaching math in a college classroom. It was very enjoyable for both him and his students.

But, now the man, the mathematician, was haunted by his high school experiences, both as a student and a teacher. He knew many students who were "failing" math could learn and benefit from math if only they could be "tutored properly" by someone.

He left teaching to pursue a business career since he was hungry for other adventures. Math remained a hobby. And, math was quite useful in many of his business adventures, one of which was teaching adult workers in technical fields.

However, he fantasized that one day there would be a way to "tutor" any high school student in a cost effective logistically practical way.

I was that boy.

Finally, that day has arrived thanks to the exponential growth of our technologies.

Thus, this book.

Craig Hane

P.S. I think this story is going to have a happy ending. I invite you to participate in it with me.

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The Problem

Chapter 1

Our "Failing" High School Math Students and Curriculum

How and Why our current high school mathematics curriculum is "failing" many of our students, and sometimes handicapping our best students.

In a Nutshell, our high school math curriculum . . .

. . . is TOO compartmentalized.

. . . does NOT utilize the Power Tools of Math.

. . . is NOT <u>interactive</u> and <u>self paced</u> enough.

Consequently, it does not meet the needs of many of our students, both those who might learn at a slower pace, and those who are accelerated. Both groups suffer.

If you are interested in the math education of a high school student, either as the student, parent, principal or the teacher, then you undoubtedly are aware of the dismal statistics pertaining to the performance of math students in the U.S. especially compared to other nations.

Fortunately, You don't have to contribute to those negative statistics. This book will explain why and how.

We all know that in today's modern economy a successful career or job in any quantitative subject requires math skills and competency, and the ability to use modern tools and technologies.

Manufactured products produced in the U.S. are increasing each year, while employment in manufacturing is declining. The U.S. is still by a large margin the world's leading producer of manufactured goods, even though many don't realize this due to the negative publicity regarding manufacturing's declining employment.

Why is this?

Manufacturing is becoming more and more productive due to the increased use of sophisticated tools and technology. Unskilled jobs, or even semi-skilled jobs, are rapidly disappearing. Today's modern manufacturing jobs require mastery of sophisticated equipment like PLC's, CNC's, and other automated systems.

This mastery depends on a core mathematical competency, in knowledge and skills.

This also applies to the fastest growing segment of the U.S. economy, health care. Ever more sophisticated

systems are being utilized which require their operators and users to be more skilled mathematically.

Science and engineering require an ever expanding mathematics skill set. Not only mastery of core math concepts and methods, but also the ability to construct and apply sophisticated math models and programs.

Our current high school math curriculum is woefully inadequate and is not serving our students.

Let me be clear at the outset.

This is not the fault of the math teachers.

Modern high school math teachers are heroes in my opinion.

The Problem is a Systemic Problem.

Why this is so, and what to do about it, is the subject of these messages delivered as chapters in this book, and as a series of Blogs at our website:

www.TriadMathInc.com

Chapter 1 is an overview look at this problem and one possible solution.

First, ". . . is too compartmentalized".

Our modern math curricula divide math into a variety of subject areas called "precalculus math" including:

Prealgebra - - - Algebra - - - College Algebra
Plane Geometry - - - Solid Geometry - - - Trigonometry
Analytic geometry - - - Finite Math - - - Etc.

This division has its roots in the 19th century math curricula and has not advanced much in the sixty years I have been involved with mathematics, as student, teacher, and mathematician.

The student is often left with the impression that these various subjects are somehow independent areas.

This is a horrible misconception, unfortunately shared by some teachers who are often forced to "specialize" on one or two topics.

These subjects are intimately related and inter-connected, as any mathematician would tell you.

It is difficult to solve many modern practical problems without invoking concepts and techniques from several of these areas simultaneously. The sooner a

student realizes this and achieves a basic mastery of several of these topics, the better.

A student needs such a Foundation as early as possible.

Let me give you a quick example.

Find the area to two decimal places of the triangle whose sides measure 3.00 ft., 4.00 ft., and 6.00 ft. The Answer is 5.33 sq. ft. This is the type of problem that could arise in a practical situation.

Looks easy, but it is not. The usual area formula from geometry is not applicable. You can solve it with algebra and geometry (simultaneous quadratic equations), but this is tedious and difficult, especially without a calculator. You could apply Heron's formula if you knew it.

A good solution is to use some trigonometry and a calculator. Then it can be solved in less than a minute by any Practical Math Foundation graduate. And, it is just as easy to find the area of any other triangle with three known side lengths. Try 13.5 in, 16.8 in., and 25.6 in. Answer. 102.9 sq. in.

In the standard high school math curriculum this problem probably would not be presented in a geometry class. Problems are rigged to be solvable with the limited tools presented there.

Give this problem to any high school math graduate or teacher and see how long it takes them, if they don't just give up.

So practical problems such as this may require up to three years of math courses to solve, and this is just one of many such examples. This is simply unacceptable considering how unnecessary it is. It's quite unfair to the student.

In a modern curriculum these various subjects should be taught in a tiered or layered manner so the student can realize their interconnections very early on, say within three months.

For example, in the Dr. Del's Practical Math Foundation, basic facts and techniques from Algebra, Geometry, and Trigonometry are all presented so the student can see the interconnections very early on, and solve many practical problems quickly and easily.

It takes all three of these subjects to accomplish this. The Practical Math Foundation takes a student about forty hours (+/- twenty hours) to complete over a one to three month period.

Now, let's address the other two deficiencies.

Second " . . . does not utilize the modern power tools of math adequately".

and

Third ". . . is not interactive and self-paced enough".

I will use the Dr. Del Practical Math Foundation described in detail later in the book to illustrate one possible solution that addresses these two deficiencies.

- The failure to adequately use the power tools of mathematics.
- The failure to give the student a self-paced and interactive experience.

Every student learns best at his or her individual pace and rate. A student may need to go over the material repeatedly until mastery is achieved. This is virtually impossible in today's "batch" teaching environment of the typical classroom.

Lectures just don't cut it for many students. Self study from books is inadequate too. Math is a 'contact sport' that requires practice where each student should advance at his or her own pace.

Mistakes should be celebrated as a sign of activity, and successes celebrated and recognized for each and every student, continually and consistently.

This is demonstrated in the Practical Math Foundation which teaches a beginning student all of the math he or she needs to know to solve many practical problems that arise in everyday life and industry. You probably have to witness it to believe it.

When the student completes the Foundation he or she will know more math than 95% of the U.S. population and be able to solve most practical problems arising in everyday life and industry efficiently using a scientific calculator.

In just three months? Yes. And, he or she will be prepared for a technical career, not however, a STEM career. More math will be required, but the Practical Math Foundation will provide a good basis for future math studies.

Very few high school graduates today know this much math. It would take at least three or four years of today's high school math to learn this much material, mixed in with a whole lot of other materials not required for everyday practical math.

This is one reason many students discontinue their math studies as soon as permitted.

Here's how the Practical Math Foundation works.

It is cost effective because it is all accomplished via online videos, targeted homework, and online quizzes, all managed by a Learning Management System. No standard textbook is used. A $10 calculator is required.

The student is given a Notebook with notes for each lesson or topic and an Exercise Book with exercises and answers for each lesson or topic.

Finally, the student is given an online quiz to let the student measure his or her success and mastery of each lesson or topic.

This is how the course works.

First, the student is taught to use a scientific calculator (the TI30XA which costs about $10). This power tool:

1. Removes much of the drudgery of calculations
2. Increases accuracy
3. Mitigates errors due to consistent checking
4. *Speeds up the learning of the concepts and techniques by at least an order of magnitude*

Second, the Foundation materials are presented in a self-paced and interactive way with continual reinforcement via homework and quizzes. This self-pacing is critical. So is the ability to review topics. Both are critical keys to the success of the Foundation.

Third, the student's psychology regarding math is "corrected" early on in the Foundation, if necessary. This is explained in Chapter 2 and the materials about the Foundation at: www.TriadMathInc.com

The Foundation can serve as a model for any teacher in any math program. The Foundation can be a good stand alone resource to supplement their program, if needed.

Indeed, home school teachers or parents can use it even if they are not math teachers themselves.

So to summarize, the “problems” with today’s high school math curricula, are:

Lack of integration of the various subject areas and presentation in a layered or tiered manner

Failure to use the power tools of math adequately

Lack of self-pacing and interactivity tailored to each student’s needs

Future Chapters address these issues in more detail and one possible modern solution to this problem is discussed in some detail.

Ultimately, any math teacher can adopt the concepts and ideas from these messages to improve their presentation of mathematics IF they are permitted to do so.

Some Facts
and
Some Ideas

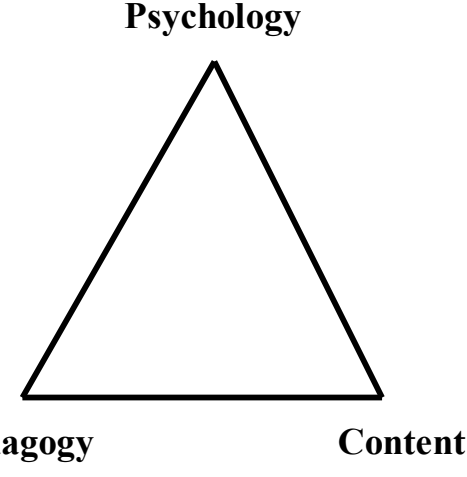
Chapter 2

The Effective Math Education Triad

There are three components to successful math education which we call the Triad.

If any one of them is missing or defective, then the math education will be subpar often to the point of being ineffective.

The 'Effective Math Education' Triad
Advice from Dr. Del, Math Teacher



For more information you may visit:
www.TriadMathInc.com

I. Psychology - - - The Most Important Component

Who's?

The student's, of course. This is often the student's most serious impediment for learning math.

Eliminate Fear and any other Negative Attitudes towards math. This should be the teacher's First order of business.

Create Enthusiasm and Joy for "doing math". This should be the teacher's Second order of business.

Embrace mistakes as learning experiences and as evidence of progress and activity. Be sure the student is always learning new things and having successes. Recognize and praise these successes.

Psychology is Job One for the math teacher. Fail at this, and all else will be for naught. So the question is: How does a teacher achieve all of this?

The answer is:

Pedagogy and Content

II. Pedagogy

Be sure you are presenting the student a topic s/he is prepared for. This is an absolute prerequisite for success.

How can you be sure? Ask Questions and practice The Socratic Technique. Be sure the student climbs the "ladder" of math understanding one step at a time. Always ask students questions to ascertain what they know and don't know.

Homework and quizzes can achieve this. DO NOT assume the student understands something you tell him or her.

Lecturing is usually ineffective and *can be* a disaster. When possible, follow up your explanation with a question that will verify the student understands.

You might use a Quiz to achieve this. Or, give a problem that will demonstrate their mastery and understanding of the topic. Homework problems can be of great help here.

Lead the student through a thought process with a series of "leading" questions when possible. This is sometimes called the Socratic Technique.

Encourage the student to ask questions. Here a Forum can be quite effective. So can interactions with fellow students.

Encourage the students to help other students. Once again, a Forum is a good opportunity for this.

Encourage the students to try things, and praise even when they do something "wrong" or make a mistake. This is a very good indicator of effort and activity.

The path to success in learning math is paved with mistakes and false starts and wrong guesses, and subsequent humor and laughter is very helpful.

Once a student has achieved understanding of a topic reinforce it with a series of problems. Then give much praise and recognition.

Avoid competition and comparisons.

Celebrate each individual achievement.

Focus on individual knowledge and skill gains, and celebrate them.

III. Content

Math is best learned in a layered or tiered manner. Learn concepts at an intuitive and practical level first. Then add additional concepts and sophistication in layers. Each time you revisit a topic is a chance to understand it at a deeper level.

For example, learn how to calculate the area of a circle first. Do a bunch of problems. Use a calculator and don't get bogged down by time consuming manual and tedious algorithms for calculations.

Later learn how to "justify" the formula for the area with an Archimedes type argument which heuristically demonstrates the area as the product of C/2 x D/2, which is, of course, then shown to be equivalent of the standard formula for area. Even later, in a future Tier, learn a more rigorous "proof" of the area formula.

A more dramatic example is the volume and surface area of a sphere.

First, present the formulas in an intuitive way to remember them like Archimedes Tombstone, Tier 2. No justification. Just remember them with a memory device.

Then, give a geometric "proof" using Cavalieri's Principle in the next Tier 3. This is presented brilliantly in Dr. Simmons book on about page 12. Later on, give a

more “calculus like” proof using math induction and an infinite series in the next Tier 4, later in Simmons book.

Finally, give a more formal calculus proof using the Fundamental Theorem of Calculus in Tier 5. Thus, the student learns the “meaning” of these formulas at a deeper and deeper level in several sequential Tiers or layers of understanding.

In fact, Dr. Del does not recommend teaching the student really formal deductive proofs until Tier 7. Too much rigorous logic too soon is a great way to “turn off” students. Worse yet, it impedes the development of math intuition and understanding of many concepts.

For example, learn to use the Generalized Pythagorean Theorem (commonly called the Law of Cosines) before you try to prove it. This is critical in understanding triangles.

Remember, Dr. Del’s Math is for high school students, many of whom will never become mathematicians and require rigorous proofs. Good reasoning is important from the beginning. But, not rigorous proofs too early in the student’s education.

When appropriate, tell the student that the “proof” for certain formulas will be postponed to future Tiers. For advanced or precocious students it is fine to give them a quick “preview”. But, then they will advance up the

Tiers very quickly due to the self pacing of the training. So they will get to the rigor soon enough anyway.

Also, be aware that any student who completes all ten Tiers will be more advanced in their 'mathematical maturity' than even many college math majors.

Any student who completes Tiers 5 and 6 will be better prepared than most of their peers when they matriculate as freshman even at Harvard or MIT. They will be ready to pursue a STEM career course of study, and should score quite high on the SAT tests.

The reason there should be ten Tiers, is that some students will be able to easily complete the first six Tiers by the end of their high school sophomore or junior year. They then will be able to begin to take college courses, if they are available. For STE career students of moderate ability or aptitude this would be the most desirable path.

Not so for M students, i.e. future mathematicians. For students who think they might like to pursue a math major, Tiers 7 through 10 would be better. Then when they go to college they probably will be ready to take junior and senior level math courses like modern algebra and topology.

This may sound too ambitious. However, today's most precocious students who have private teachers or tutors do this, especially in non-U.S. countries.

You should be aware, that calculus and differential equations are considered basic courses by mathematicians. Modern abstract algebra is often the first advanced course math majors take and they are "weeded out" fast if they are unprepared.

If a student anticipates a career as a theoretical mathematician, the Ten Tiers will be a much superior route to take. Much more important and difficult than learning "deductive reasoning" or "proofs" in Tier 7, is learning certain abstract mathematical concepts and structures in Tiers 8 and 9.

Exposure to "quotient structures" in Tier 8 will give the student a few years "head start" in pursuit of a math major career. Then, learning about modern abstract math structures in Tier 9 will give him or her an even greater head start. Tier 10 is like "icing on the cake".

This is the type of math education that can prepare a precocious student to embark on a graduate career in theoretical math in their late teens, which is quite desirable for those rare students who will pursue this career path.

Dr. Del didn't learn many of these things until he was in graduate school in his early and mid twenties. He realized then that he was many years behind the math leaders of his day. Just read Silvia Nasir's "A Beautiful Mind" and consider John Nash's career, or John Milnor's, or John Tate's. Don't bother with the movie. Read the book.

We owe it to our very brightest math students to provide them with the advantages they will need to compete with their brightest and best educated peers from around the world. You may be sure that in some other countries their best math students will be given this type of math education.

Virtually no U. S. high school math teachers have the background to offer this to even their best students. That's not meant to be a criticism, just a fact of life.

In fact, most students can't get to this type of training until late in their undergraduate career, or early graduate career. And, then it is almost too late. It would be like starting to take serious piano lessons in one's twenties. Awfully late.

It would be analogous to not beginning to seriously practicing a sport until late in one's teens. Then it is almost too late to become a star or even to excel.

Math is a lot like a sport. The younger you start the quicker you will gain maturity and knowledge and be able to excel. And, if you start too late it might never happen. See Malcom Gladwell's book *Outliers*.

The learning curve for math is like most learning curves, exponential in nature. Thus, even a year or two head start can mean a great deal in graduate school. For an amateur or STE major, it doesn't much matter.

However, for a future professional mathematician, just like a future professional athlete, it really does matter. That is why Dr. Del is dedicated to presenting the top four Tiers, 7 thru 10, to those *few rare students* who really need them. Besides, it's what he loves to teach the most. But, they won't be ready until they have completed the first six Tiers, which he loves to teach too.

A Special Message for High School Math Teachers.

IF you are a high school math teacher, YOU owe it to yourself and to your students to fully understand the contents of this message. Dr. Del hopes that his materials will benefit math teachers at all levels as much as it benefits students.

Indeed, Student/Teacher are two sides of the same coin. Dr. Del believes the students will benefit most

from his materials if they have a teacher whom they can discuss math with.

Math is an infinitely rich subject. No one will ever know it all, or know it as deeply as possible. Just think Riemann Hypothesis. Dr. Del continually learns from his students as they are learning from him.

In fact, Dr. Del believes he never really understands a topic until he has taught it, usually several times. So, Dr. Del encourages all of his students to also become teachers of other students. It might just be in a help group or on a forum or helping a friend.

Teach/Learn. Two sides of the same coin.

Dr. Del very much wants his Math Program to be an inspiration and resource for other math teachers.

Dr. Del's motto is:

Never be Satisfied, Always be Content

He believes Math is one of the greatest "games" ever invented by man. One is never satisfied with what math they have learned or the problems they have solved. Each leads to the next one.

However, it can be a great contentment to learn, and teach, math and to solve the next problem, or learn the next concept, or prove the next theorem.

So, students and teachers alike:

Come on in, the water's fine.

Splash around and have fun.

You won't drown.

Guaranteed.

Who knows what distance shores you might swim to? Math, like life, is an adventure, and can be great fun as well as rewarding in many ways.

Indeed, becoming "matherate" just might be the student's ticket to great economic success in this very different 21st Century Third Wave Economy as explained in Chapter 8 and page 133.

Chapter 3

Power Tools of Modern Mathematics

There are Four Power Tools that make the learning and practice of mathematics amazing in this modern 21st Century world we all live in.

Failure to utilize them appropriately and optimally in both the learning and practice of mathematics can render our children Second Class citizens in this new Third Wave, Flat World.

Who is the best chess player in the world? Is it a human? Some years ago the answer would have been yes. Is it a machine? A few years ago the answer would have been yes. Answer: Neither of the above.

Today the best chess player in the world is a combination of human and machine. That is a good chess player (not a grand master) and a good machine (not nearly as powerful as IBM's Big Blue) working together. At least this was described in an article I recently read about an open international chess tournament.

Today, in many fields the most powerful "player" is a human/machine combination. In industry, think AutoCad, CNCs, and PLCs. In health care, imagine a doctor without his diagnostic machines and powerful information systems.

Imagine a carpenter without his or her power tools. Or a farmer without modern agriculture machines. Or, ... pick your favorite topic.

In most fields there are powerful machines which amplify the abilities of a human resulting in a superior performance to any pure human or pure machine.

This is quite true of mathematics. In the last forty years there have been remarkable "machines" or "power tools" developed which greatly amplify the abilities of any human mathematician in both pure and applied mathematics.

Modern mathematics education must take this into account in the teaching and learning of mathematics if it is to be optimal for our students.

In this Chapter I will describe four remarkable tools any aspiring mathematician should know about. Of course, there are many more, and more being developed each day. We are all on a very "fast track".

Failure to utilize these tools in both the teaching, learning, and practice of mathematics will render our modern workers impotent.

Just imagine modern life and jobs without these tools just as you might imagine it without the Internet or cell phones or computers, etc., etc.

The Scientific Calculator (1972)

The first is the scientific calculator which tremendously amplifies the ability to learn and perform mathematical calculations, and thus solve problems. The first one was the HP-35 introduced in 1972. It sold for $395 (about $2,000 in 2011 dollars).

http://en.wikipedia.org/wiki/HP-35

I was teaching at an engineering school when this happened. It was both inspirational and traumatic for both the math and engineering faculty. It virtually revolutionized the way math could be applied to engineering.

It didn't actually affect me much since I was teaching the advanced level math theory classes. But, I really enjoyed dialoging with "confused" colleagues who were going to have to drastically revamp and revise their curricula.

At that time all numerical calculations were done manually utilizing extensive tables such as trig tables and logarithms. Many tedious and laborious algorithms had to be mastered.

The most sophisticated tool was the slide rule which is based on logarithms. *An engineer could spend 90% of*

his time performing the myriad numerical calculations in solving a problem. Suddenly this was reduced to 10%.

This greatly freed up time to devote to more conceptual aspects of math and problem solving. Not to mention the reduction of errors.

WOW. An increase of both productivity and quality. And, a dramatic lowering of labor costs.

Of course, it completely changed the curriculum. Some professors resisted by banning calculators with the excuse they were too expensive. But, the price quickly came down and slide rules and the many tables became obsolete along with many of the manual algorithms.

Never again would one have to calculate a square root manually. Never again would one need to perform calculations using logarithms. Slide rules were history.

Soon many other types of calculators were developed and the prices dropped. Today, in 2011, I use a scientific calculator, the TI 30XA, which is much more powerful than the HP 35 and costs only about $10.

In our Dr. Del's Practical Math Foundation we teach the basic ideas and facts about Algebra, Geometry, and Trigonometry needed to solve most practical everyday problems in about 40 hours of self-paced student time.

This is only possible because we use the calculator for all the tedious and laborious calculations.

The scientific calculator didn't have too much effect on business. Then came . . .

The Spreadsheet (1979)

In 1979 came the first spreadsheet, VisiCalc, which ran on the Apple home computer. WOW.

http://en.wikipedia.org/wiki/VisiCalc

The spreadsheet revolutionized the way business and accounting math was done. Again tedious work was reduced by at least an order of magnitude, probably many orders of magnitude when one considers the "what if" games one could now engage in.

Can you imagine teaching accounting today without spreadsheets? Ultimately Excel, and its free open source version, became the standard. Today many math problems and models can be solved utilizing Excel.

The spreadsheet is an indispensible tool for modern mathematicians in the business world. In fact, it has applications in many other arenas.

For example, Statistical Process Control (SPC), which is the basis of modern management techniques like

Six Sigma and Lean Manufacturing, and modern Quality Control, used to be very laborious.

One had to first collect the data and then construct SPC Control Charts. In fact, one could spend 90% of one's time just constructing the SPC Control Charts, and this was difficult and tedious to learn. Thus, SPC was relegated to specialists.

Then in 1998 *SPC for Excel* was introduced:

http://www.spcforexcel.com/home

This remarkable tool eliminates all of these manual Chart constructions. Now one can spend all of one's time collecting and analyzing data and understanding the process.

NO chart constructions today. It is all done for you. In fact, it can create charts impossible to do manually

You see, Excel, is also a powerful "programming language" via what are called "Macros". And *SPC for Excel* is just a collection of such programs that now do what used to be done manually. Such tools revolutionize how SPC is done. It opens SPC up to non-specialists.

There are many other such tools in many other areas.

The Best was yet to come!

In 1988 a truly remarkable tool was created by a young genius after he won the MacArthur Award.

Mathematica (1988)

A remarkable tool called Mathematica was unleashed on the world. Things have never been the same since then. WOOOOW!

http://en.wikipedia.org/wiki/Mathematica

Mathematica does for calculus and differential equations and linear algebra, and many other areas of mathematics what Spreadsheets did for SPC and business.

I am still amazed by Mathematica. It's like magic. Thank you Steve Wolfram.

Using Mathematica, now one can learn the concepts of calculus and differential equations very quickly. Mathematica does all of the amazing calculations and graphing including difficult and tedious differentiation and integration calculations, and much more.

Many of the manual calculus calculation techniques taught in calculus are now obsolete the same way logarithms for arithmetic calculations are obsolete.

These calculus calculation techniques took up 50% or more of the student's time, and were boring and tedious and difficult and time consuming to learn and master.

In fact, "Techniques of Integration" in Calc 2 probably flunked more kids out of engineering school than anything else. These are a bunch of ad hoc techniques, via antiderivatives and the Fundamental Theorem of Calculus for evaluating integrals, that are difficult to master. Now obsolete. Yet still taught in many calculus courses, because of tradition I suppose. A shame?

It is now possible to teach the concepts of Calculus and how to apply them in a just few weeks or months. The student's understanding is actually enhanced by removing the tedious drudgery of manual calculations, just as it is in the Practical Math Foundations course for practical math.

The same thing applies to Differential Equations. Of course, calculus and differential equations are the workhorses of modern STE subjects, along with linear algebra and few other subjects to a lesser extent.

Now Mathematica removes the drudgery and amplifies our time by orders of magnitude. No one in their right mind would do a difficult manual integration now.

Even more significantly, Mathematica is really a very powerful programming language that can be used to create amazing sophisticated math models.

We believe the sooner a math student is exposed to Mathematica the better.

We will expose them to it in Tier 4 and use it extensively in Tiers 5 and 6 when those programs are developed as planned. This is what we propose for the modern high school math curriculum.

However, Mathematica is pretty expensive. Then, along came a remarkable new free online version of Mathematica that has now been turned loose on the world.

This new power tool is free and should revolutionize the way we can teach calculus and differential equations to high school students.

Historically, learning the concepts and applications of calculus has required learning many difficult and tedious "calculation" techniques. Derivatives aren't too bad, but techniques of integration are quite difficult to master.

Ironically, the concepts of both calculus and differential equations can be learned quite easily in a short time. It is the calculations that take so much time and effort to master.

Now this new free tool does for calc and diff eq what the scientific calculator has done for arithmetic calculations. *We simply must take advantage of it in our modern high school mathematics curriculum.*

Wolfram Alpha (2009) Revolutionary.

What is the 1,000th pair of twin primes?

Answer: 79,559 and 79,561

How long would it take you to find this out? It took about ten seconds with Wolfram Alpha.

What is the integral of SIN(x^2 +1) from -1 to 1?

$$\int_{-1}^{1} \sin(x^2+1)\,dx = \sqrt{2\pi}\left(C\left(\sqrt{\frac{2}{\pi}}\right)\sin(1) + S\left(\sqrt{\frac{2}{\pi}}\right)\cos(1)\right) \approx 1.85754$$

Twenty seconds.

In 2009 Wolfram Alpha was introduced to the world. Thanks again Steve.

http://www.wolframalpha.com/ &
http://en.wikipedia.org/wiki/Wolfram_Alpha

Stephen Wolfram is the creator of Mathematica, and Wolfram Alpha is now a free online version of Mathematica that is quite adequate for our use in the Dr. Del math program.

It is truly amazing.

One last example: Area of triangle with sides 3, 4, and 6? In about five seconds

$$\frac{\sqrt{455}}{4} \approx 5.33268$$

In the Practical Math Foundation Course we learn to do this with a TI30XA calculator in about one minute. And, of course, we have a better understanding than just the answer.

We use the Generalized Pythagorean Theorem and the Corrected Triangle Area Formula. But, it's great to be able to check it quickly.

Try it manually without a "machine" or power tool. Hint: You can use Heron's formula or simultaneous quadratic equations derived from the Pythagorean Theorem, or the Trigonometry way using trig tables.

Any purely manual way is a tedious "nightmare". That's why I like to give this problem as a "mini-test" to see if a student or teacher will benefit from the Foundation Course. It's the last thing we teach in the Foundation Course.

See the Interlude #3. (Five video lessons)

This Interlude should give you a good idea of what I am recommending for the Tiered treatment of the teaching of mathematics.

The five videos span Tiers 2 thru 4, with some extension material that can be used to challenge the best students.

MODERN IMPERATIVE (2011)
for High School Math Education

We must begin to train our students to use these "power tools" from an early age. Using these tools is like using a musical instrument or sports equipment. The sooner one begins to learn about them and to use them the better.

I believe one should use a simple $1 calculator as a teaching tool to help very young students learn the addition and multiplication tables. Make it a game. And, then to learn the meaning of the decimal number system. And, then how to do approximate calculations.

But, certainly by high school math one should begin to use these modern power tools.

We use the TI 30XA Scientific Calculator in the Tiers 1 and 2. It is one of the three "secrets" to the success of the Practical Math Foundation Course.

We introduce Spreadsheets (Open Source) and graphing in Tier 3. We introduce Mathematica in the form of Wolfram Alpha in Tier 4, and use it extensively in Tiers 5 and 6.

However you choose to educate your students in mathematics, you should take these power tools into

consideration and integrate them into your educational program.

You may be sure our international competitors are doing so, and the future workers who are most successful will be those who are very capable of amplifying their thought processes with “machines”.

Math educators who advocate not using calculators or other more advanced tools in the teaching of high school math are, in my opinion, just ignorant of the realities of today’s workplace and the proper interfacing of technology into the learning and practice of mathematics.

Furthermore, I believe it is much easier for a student to learn the concepts of math and the applications of them to problem solving if relieved of the horrible burden of manual calculations. Indeed, I know so.

Today, I can teach virtually any subject from prealgebra to trigonometry in a small fraction of the time it took prior to 1972, and calculus and differential equations in a small fraction of the time it took prior to 1988.

To ignore these modern imperatives of our technologies and economy does a great disservice to our students and will put them at a serious handicap compared to students who are properly educated.

Doing math without the power tools would be like doing carpentry without the appropriate power tools like nail gun and table saw and drill.

The power tools of mathematics can make math more like a modern game, which it is. Thus, math can and should be fun and exciting. These modern tools greatly contribute to this.

Simple mastery of the TI 30XA will let you perform wonderful "magic like" calculations and problem solving. I believe learning modern math should be as much "fun" as playing video games, or chess, or any sport. And, the potential dividends are much greater.

Mathematics is the KEY to unlocking deep understanding in science and technology, and economics and philosophy.

Personally, I believe it may be the key to deeper understanding of metaphysics via such works as those of Dr. William Tiller. Dr. Tiller's Maxwellian type equations for the non-physical universe is a tour-de-force comprehensible only with mathematics.

Indeed, understanding the existence of a non-physical universe which coexists with our physical universe requires understanding the mathematics of Einstein's Theories and Quantum Theories. All explained by Dr. Tiller in his books.

But, back down to earth.

An understanding of practical mathematics such as taught in Tiers 1 and 2 should be prerequisite for any human being in this modern world. Anything less is like being illiterate, or "immatherate".

Definitions in The Free Dictionary by Farlex

I. lit·er·ate
adj.
1. a. Able to read and write.
b. Knowledgeable or educated in a particular field or fields.
2. Familiar with literature; literary.
n.
1. One who can read and write.
2. A well-informed, educated person.

II. math·er·ate
adj.
1. a. Able to solve math problems.
b. Knowledgeable or educated in mathematics.
2. Familiar with mathematics.

n.
1. One who knows and can use mathematics
2. A well-informed, educated person.

I. is in this dictionary.
II. is not, , , , yet.

Interlude #1 Triangle Problem #1

These Interludes are for math teachers who already understand mathematics.

A triangle is a "rigid" polygon. It has three sides whose lengths determine the size of its included angles, which add up to 180^{o} in Euclidean Geometry.

Polygons with four or more sides are not rigid, in the sense that the lengths of their sides do not determine their angles. That is one reason triangles are so important in construction and practical math. It is imperative that we know all about this most important polygon, the triangle.

So, we teach the student everything he or she will ever need to "solve" any triangle problem. That means given adequate information about a triangle we can calculate all other things we want to know about it.

So, here is the first of three examples of things we should teach beginners in their first pass through high school math in the Foundation.

Suppose you have a triangle with two sides measuring 12.5 U and 18.6 U and included angle 49^{o}. What is its area? You might want to draw a picture as we talk.

Why don't you figure it out and think how you would teach a student? We all learn an area formula for triangles as:

Area = .5xBasexHeight.

But, the Height is not given here.

Now, .5x12.5x18.6 = 116.3 U^2
would work IF this were a 90^o angle. But it's not.

So we need a Correction Factor, CF.

A CF that works in this case is .755,

So the Area = 87.8 U^2

Of course, you can find this CF in about five seconds with any scientific calculator today, which we teach the student to do. Indeed, the CF = SIN(49^o)

Visit www.*TriadMathInc*.com/m17 for details of how we teach it.

Remember, these Interludes are for math teachers.

Chapter 4

Mathematics Educational Resources

This Chapter is for both students and teachers who are not satisfied with the materials they are using to learn and teach mathematics.

This can be homeschoolers, tech school students, frustrated parents or students, and even regular high school teachers.

Unfortunately, I can not recommend any of the modern math textbooks for reasons that will be clear to anyone who is familiar with the Dr. Del's Ten Tier system and the Effective Math Education Triad.

Indeed, only the first six Tiers will apply as a model for the new high school math curriculum I am proposing.

There is one text book I do recommend and that we use in Tiers 3 and 4. It is Dr. George Simmons, *Precalculus Mathematics in a Nutshell.*

http://www.amazon.com/Precalculus-Mathematics-Nutshell-Geometry-Trigonometry/dp/1592441300

It is not fancy. It is only 119 pages long. It costs about $20. It is terrific.

If a student could learn everything in this book he or she would be better prepared than most matriculating

freshman at the best universities, and well prepared for calculus.

We plan to use Dr. Simmons' book in Tiers 3 and 4. We will cover about 80% of it in Tier 3. And, then we will cover the remaining 20% in Tier 4 along with much more material such as Complex Numbers and Spreadsheets. See Chapter's 5 for elaboration.

PreCalculus Math in a Nutshell is not very good for self-study for most students since it is very condensed with no fluff, as Dr. Simmons says. It is great with a good teacher and I highly recommend it if you are a competent high school math teacher.

There are many resources for math education available today. Some are books and DVDs that follow a fairly conventional high school math curriculum.

I consider most of these resources of little value, and indeed potentially dangerous since they often "turn off" many students and are difficult to teach from.

It isn't the fault of the authors. They are just trying to satisfy the demands of the standard high school curriculum. That is where the problem lies that I discuss copiously in other Chapters.

I would like to tell you about some resources I personally use and like, as well as one I have created for certain students and teachers.

A. The Khan Academy

The Khan Academy is the creation of Sal Khan.

http://www.khanacademy.org/

It is an incredible collection of YouTube videos on many subjects and topics including mathematics. It has many fans and users including Bill Gates. It is free.

Whenever I am interested in a topic in chemistry, physics, history, economics or even something I don't know about math, I go there and try to find a video. Khan's videos are short and pretty informal and quite good.

Khan says that some of his students prefer his videos to him personally tutoring them. This is quite significant, pedagogically speaking. We can all learn from it.

When a student is taught in a classroom or tutored there is always a pressure to press ahead and not "waste" time. The student is on the teacher's time table. But, the student needs to be on the student's time table.

Many students are afraid to ask questions, or to ask for something to be repeated. *With videos this serious pedagogical deficiency is avoided.*

Videos can be paused, backed up, and replayed until the student masters and understands the material, thus self pacing and interactive.

There is no doubt that one of the best ways to teach and learn math is via videos that can be self-paced by the student and watched repeatedly at the student's convenience. And, reviewed periodically which is important too. That's what Khan's students taught him, and all of us.

Khan's math videos seem to me to follow pretty much what I would say is a standard math curriculum. I suppose it's the way Khan learned math and the way math is taught in most high schools today. If you are teaching or learning from a standard typical math course, then Khan's videos can be a great resource.

That said, I think the standard curriculum is the wrong approach to math education for many reasons I give in other Chapters.

I do not consider the Khan Academy in any way competitive with the Dr. Del Ten Tier Math Program.

Indeed, I can not see how one could achieve even the Practical Math Foundation goals with Khan's videos for many reasons. But, they are still a great resource for any student needing help with a standard math course.

B. Wikipedia

Wikipedia is an incredible resource for just about any topic you can think of. It's free too.

http://www.wikipedia.org/

Just go there and search for any topic you wish to learn about. Unlike Khan Academy, Wikipedia is almost all text based. If you just want to learn some facts, this is often the best way to learn them quickly. If you are trying to learn some concepts or techniques, then video is better.

For example, one of the things I emphasize is the necessity of including a proper treatment of Complex Numbers in the high school math curriculum very early on. You might wonder why.

We will teach the Complex Number System in Tier 4, utilizing what I think is the most important equation in all of mathematics, which expresses the equivalence of the algebraic and geometric definitions of complex number multiplication. See the last two Interludes, #5 and #6, for more info on this.

One reason is that Complex Numbers are used in many science and engineering fields. It is easy to learn Complex Numbers when they are taught appropriately.

On the next page are just a few of the fields they are used in, from Wikipedia.

http://en.wikipedia.org/wiki/Complex_number

 7 Applications

- 7.1 Control theory
- 7.2 Signal analysis
- 7.3 Improper integrals
- 7.4 Quantum mechanics
- 7.5 Relativity
- 7.6 Dynamic equations
- 7.7 Fluid dynamics
- 7.8 Fractals
- 7.9 Algebraic number theory
- 7.10 Analytic number theory

Having said this. Wikipedia is NOT a good place to really learn about Complex Numbers.

The Dr. Del videos in Tier 4 will be much better for a variety of technical reasons you will appreciate after learning about complex numbers.

The Complex Number System should be introduced properly to a math student during the pre-calculus phase. It will shed deep light on the trigonometric functions, and algebraic equation solutions.

In fact, it is how I remember, and derive quickly, all of the major trig identities. It fulfills the understanding of quadratic equations and much more, such as the roots of unity.

C. Dr. Del's Ten Tier Math Program

The Dr. Del Ten Tier Math Program is under development. The first two Tiers are completed. So are Extensions for what we call the ECA/COMPASS exams.

The Plans and Syllabi for the other eight Tiers are presented in this book in Chapters 5 and 11, and are also discussed on the Website:

www.TriadMathInc.com

I will use the Dr. Del Ten Tier Program as a model of what I believe any modern high school math curricula should consist of. The Ten Tier Program will be an online program that will achieve the three objectives of the Triad of Effective Mathematics Education. (See Chapter 2).

It will consist of Ten Tiers, the first two of which are already finished and being used successfully.

The vast majority of students will only go through Tier 6 before matriculating to some university.

The first two Tiers comprise what is called the Dr. Del's Practical Math Foundation and will be sufficient for those students wishing to pursue a non-professional career in some technical subject.

For example, today there is the opportunity for a worker to become an electrician and earn a really good living. The Practical Math Foundation math will provide an excellent math foundation for this type of career.

The same applies to many other technical service areas, which will be in great demand and filled by non college graduates.

Tiers 3 and 4 will present a very good grounding in what is called precalculus mathematics and prepare the student for calculus anywhere.

Tier 5 will teach calculus using modern tools such as Wolfram-Alpha and Spreadsheets. Tier 6 will teach differential equations using modern tools. Tiers 5 and 6 are the way I believe calculus and differential equations should be taught today in an optimal high school curriculum, college too for that matter.

I believe it is unconscionable to teach these subjects the classical way for many reasons too deep and complex to discuss in detail here. See Chapter 5.

Upon completion of Tiers 5 and 6 a student will be well prepared to pursue a STE (Science, Technology, and Engineering) career at the best colleges or universities.

Tiers 7 through 10 are planned for those students who might wish to pursue a career in mathematics (M). This will be unique today at the high school level, but I hope will become the norm at some time in the future.

Interlude #2 Triangle Problem #2.

Suppose you had a triangle with two angles 57^o and 38^o and the side opposite the 57^o angle measured 23 inches. What is the length of the side opposite the 38^o angle? Call it L.

In Geometry we teach the students to use ratios of corresponding sides of two similar triangles to solve certain problems involving lengths of sides. But, here there is only one triangle.

So how do we get a ratio between L and the side measuring 23 inches? Draw a picture. Clearly L should be less than 23 since it is opposite a smaller angle.

But what? Of course, it turns out to be the following:

$L/23 = SIN(38^o)/SIN(57^o) = .616/.839 = .734$
So, $L = .734x23 = 16.9$ inches.

The so called Law of Sines is very easy for a beginning student to grasp and understand when s/he is properly prepared. And, of course they find the other angle from what they learned in Geometry, $180^o - 57^o - 38^o = 85^o$. And, now they find the length of the other side to be 27.3 inches. For more info:

www.*TriadMathInc*.com/m18

One Solution to the Problem

Chapter 5

A Modern 21st Century Math Curriculum

However it is delivered, what I now recommend to you is what I believe should be the content of an excellent high school math curriculum. For most high schools, this can only be achieved utilizing materials available via the Internet for reasons that should become apparent.

The Dr. Del Ten Tier Program is under development. Tiers 1 and 2 are completed and being used quite successfully by students today (2011).

The Plans for Tiers 3 thru 10 are also completed and presented to you here as the model for a modern high school math curriculum that will meet the needs of virtually all students, from those wishing to pursue a non-professional technical career, to those who will pursue a professional STEM career including professional mathematics.

The Six Tiers are described in some detail in this Chapter 5. Tiers 7 thru 10 for future professional mathematicians are described in Chapter 11.

Fuller details are available at:

www.TriadMathInc.com

Tier 1: A Basic Familiarization with a Scientific Calculator, the TI 30 XA, and Pre-Algebra.

This series of lessons has the objective of getting the student familiar with, and comfortable with, using this scientific calculator. It is not intended to teach very much math at this point in time.

In Pre-Algebra we explain the basic arithmetic functions and review number concepts, but do not explain any of the more advanced functions such as the trig functions.

We will have the student become familiar with most of the keys and normal arithmetic operations. They will learn to fix the decimal to so many significant digits, to use the memory and parentheses, fractions, and some of the special function keys, etc.

The goal is to get the student started learning math and comfortable with the calculator.

And, perhaps to have a little fun which is always a good thing. Fun and Games.

The scientific calculator will enable the student to learn the topics in Tier 2 in a small fraction of the time it takes in the classical approach to these subjects, and to solve more realistic problems, i.e. real world practical math used in many technical fields.

Tier 2: Dr. Del's Practical Math Foundation

Tier 2 includes the topics from Algebra, Geometry, and Trigonometry that are needed to solve ordinary practical everyday math problems. It sets the foundation for all that is to come in the upper Tiers. It is a prerequisite requirement for the upper Tiers.

A student who already knows this material will be able to complete the foundation course in less than twenty hours.

It calibrates the student's psychology for future math studies, if that is needed. And, it is for many students.

Tier 2 includes no theory to speak of, few proofs, and not many definitions. Just the basic math useful for every day life in almost any activity.

Visit: www.TriadMathInc.com for details.

The student is taught through a series of lessons each of which includes an online video to accompany the notes and exercises sets and quizzes. It is wholly self-contained and does not require a math teacher.

Virtually all students, regardless of aptitude and ability, should be able to successfully complete this course.

The average student should take 40 hours +/- 20 hours spread over one to three months to complete this course. It is self-paced, and interactive in several ways.

Some students may choose to stop here depending on their educational and career aspirations. **At this point they will know more math than about 95% of the U.S. population. Hard to believe perhaps, but true.**

Any student who can play complex games of any kind will be capable of doing well in this Practical Math Foundation program.

For a complete Syllabus of the Foundation you may use as a guideline for your own teaching visit:

www.TriadMathInc.com/Syllabus

There is also an Extension Course of this Foundation course to assist those students who need to pass an ECA exam, or to do well on the COMPASS test. This consists of selected topics that are covered in Tier 3, but for students who want to go to tech school or enter an apprentice program for a "non-professional" technical career and do not want to go on in their math studies.

Tiers 3 and 4, which are under development (2011), will be for students who want to continue their math studies to pursue a "professional" career in some STEM subject.

Tier 3: Algebra I, Geometry I, and Trigonometry I

For students who wish to pursue a STEM career in Science or Technology or Engineering or Mathematics, Tier 3 is closer to a conventional high school curriculum.

Algebra I, Geometry I, and Trigonometry I will be presented in a series of videos which yield self pacing and interactivity.

Dr. George Simmons book, *Precalculus Mathematics in a Nutshell* will be the "textbook" for these three courses. Problems will be assigned from this book. Quizzes will be on-line along with the videos just like Tiers 1 and 2 in the Foundations Course.

While self-paced, most students will complete Tier 3 in about six to nine months, a radical departure from the standard high school curriculum in most schools today.

I want to emphasize that today (2011) Tier 3 is in development and planned to be released in early 2012. However, any good high school math teacher could teach this material from Dr. Simmons great book.

Just don't do the most difficult materials in the first pass. For example, in Interlude #3 I discuss the situation with triangles and give you some guidelines and thoughts on this. I will postpone about 20% of the most difficult material until Tier 4. Then, in Tier 4 I will also review the material covered in Tier 3 quickly.

Tier 4: Algebra II, Geometry II, and Trigonometry II, plus other important topics.

This will be math presented to the student who wants a deeper understanding. It will include a few topics from so-called Finite Mathematics.

Spreadsheets will be introduced as a power tool to solve many problems and build models. This is a great way to understand exponential growth, and the use of semi-log charts. More theorems will be proven.

But, most important of all, Complex Numbers will be explained and their relationships to many other mathematics subjects, particularly trigonometry and algebra, will be discussed thoroughly.

This is important and unusual at this stage of high school math education. This distinguishes the Dr. Del Six Tiers program from all others.

Lack of adequate treatment of this crucial number system is the gaping hole in the standard math curriculum in the United States that should be filled by any new high school mathematics curriculum.

Indeed, we will be developing this treatment even before we embark on Tier 3. This is so it will be available to any high school teacher who might be able cover the Simmons book on their own, but would need help on complex numbers in the event Tier 4 is not done by us.

Tier 5: Calculus, Differential and Integral

Calculus will be explained conceptually, and the great tool Mathematica utilizing Wolfram-Alpha will be used for calculations.

This will not be "your father's course" in calculus. The important concepts and applications will be covered thoroughly. The student will be ready for engineering and science courses in college.

However, the many usual "manual" computational techniques which are so burdensome to both students and mathematicians will be omitted, thanks to Mathematica.

This is really a "big deal" for many reasons, pedagogical among the most important. This will be discussed more extensively at: www.TriadMathInc.com

All too often the important concepts of calculus are obscured by the myriad details covered in a typical course. It must be emphasized that much of the traditional materials taught in a typical calculus course are mitigated by tools such as Mathematica.

No applied mathematician in his right mind would perform a difficult old fashioned manual calculus calculation given today's tools, at least not on my payroll.

Tier 6: Differential Equations

Differential Equations (Diff Eq) are the natural extension of calculus and are the "work horses" of modern science and engineering models.

Diff Eq are about equations containing derivatives and integrals of various orders. Some are fairly easy to solve with conventional manual techniques, others not. Guess which ones arise most often.

Diff Eq will be introduced, again with Mathematica as the power tool, and what a power tool it is! It makes quick work of all Diff Eq, even those intractable by standard manual methods

This too will be a conceptual treatment that will prepare the student for a more conventional university course and STEM courses in other fields.

All but the most precocious students will stop here, and probably matriculate on to a university. They will be better prepared than all but a very few of their peers. They will have a high level of confidence in their abilities.

They will be well prepared for any STEM courses in college. They will score high on SAT type tests.

Here are some more details on the timing of this Six Tier program.

Six Tiers Timeline

I remind you that only Tiers 1 and 2, plus their ECA and COMPASS Extensions are available today, 2011. So those timelines are established from experience.

Furthermore, the planned and anticipated timelines for the Six Tier system will depend on the particular student since each Tier is designed to be taken in a self-paced manner.

It is important that each student absorb and understand the material in a way comfortable for him or her. If a student gets through Tier 6 by high school graduation he or she will be far ahead of his or her contemporaries from typical schools, both public and private.

A student may begin this process as early as their seventh grade for some students.

Tier 1 should take a student one to three weeks. It will be best if the student works all the homework exercises and studies each video as many times as necessary.

If a student already knows the material, s/he may simply take the online quiz and move on. Or, do the

homework and then take the quiz. Watching a video will be necessary if the material is new or unfamiliar.

It is important to not let too much time elapse between study sessions. Ideally the student will complete this course in about one to three weeks**.**

It is very important they use the TI30XA scientific calculator.

Tier 2 should take most beginners about two or three months.

The total time studying and mastering the material should consume anywhere from twenty to sixty hours applied in about one hour segments.

There should be NO 'reward' for finishing quickly.

Once Tier 2 is completed the student should have a considerable amount of self-confidence and no fear of math.

It is important that the teacher/mentor work with the student to be sure he or she is working steadily and understanding the material.

This will best be accomplished by the student working the exercises and taking the online quizzes. If the student is having difficulty with a concept be sure the student works the relevant exercises and watches and studies the videos. If that isn't enough have the student seek help from fellow students, or the Forum.

There should never be any embarrassment from having to go over a topic multiple times and ask questions and make some mistakes working the problems. This is normal for virtually everyone on some topic or other.

At this point the student will be able to solve almost any practical problem that comes up in ordinary everyday activities. The Foundation "graduate" will know more math than most U.S. adults.

Some students may choose to stop here. They will be better prepared for most non-professional technical jobs or careers than 95% of their peers.

However, those who want to go to university or study science or engineering will want to go on to Tier 3 and then Tier 4.

Tier 3 will consist of three courses Algebra I, Geometry I, and Trigonometry I as described above.

This program should take about six to nine months for most students. Lot's of problems and quizzes are the keys to success.

Tier 4 Algebra II, Geometry II, and Trigonometry II. Plus other topics as described above.

Tier 4 will take about one school year, or nine months for most students. But again, due to self pacing the most important thing is for the student to work steadily and enjoy the successes, and setbacks as s/he learns the materials.

Tier 5. Learning the basic concepts of calculus and an introduction to Mathematica should take about three to six months for most students.

Tier 6. Learning the basic concepts of differential equations and a continued introduction to Mathematica should take another six months for most students.

Students who complete the first six Tiers will be quite well prepared for a university career in any STEM subject. They will be ready to apply calculus and differential equations to engineering and science classes.

Here is a possible timeline for a typical student who might wish to pursue a STEM career at a good university.

Year 1		Year 2	
T1 T2	**T3**	**T3**	**T4**
3 Mo	**6 Mo**	**3 Mo**	**6Mo**

Year 3		Year 4	
T4	**T5**	**T5**	**T6**
6 Mo	**3 Mo**	**3 Mo**	**6 Mo**

NOTE: The Dr. Del Ten Tier Program is self-paced.

Students will go through the program at varying speeds depending on their particular factors.

A fairly unmotivated or weak student could take four years to complete Tiers 1 through 3. That might be just right for this student.

Another very motivated and strong student might complete all ten Tiers in four to six years. The important goal is to let each student reach his or her potential and celebrate this success.

We are all different and we learn at different rates depending on complex motivating factors. There are quick learning "prodigies", and slow learning "late bloomers".

The important thing is for each student to reach his or her potential and achieve his or her ambitions. Students may take periodic breaks for many reasons.

Learning is not linear and continuous. That is why self pacing and review is so important. More students will achieve their goals successfully in this way, than those who are forced to "lock step" to an artificial timetable for the convenience of their school or teacher.

This is what a successful 21st Century High School Mathematics Curricula must address and accommodate.

Interlude #3 Triangle Problem #3

What is the area of a 3, 4, 6 triangle? Answer $5.33U^2$

This is a deceptively difficult problem. In fact, it is the "one question test" I give anyone to decide if they will learn anything from the Practical Math Foundation.

Try it. If you can solve it in less than five minutes you know a lot of practical math and probably will find the Foundation only of value as review and ideas for content and pedagogy.

After the Foundation a student should be able to solve it in a minute or less using a scientific calculator.

Here's the problem it poses. You don't' know any of the angles, and, thus, you don't know any Height. There are some ways to solve it using Heron's formula or setting up two simultaneous quadratic equations to solve.

But, I prefer to use trigonometry. First, IF you knew the angle, <A, between two of the sides, say 3 and 4, then you could use the formula you learned, or already knew, from the first Interlude #1.

That is Area = .5x3x4xSIN(<A), CF was SIN(<A) Here $<A = 117.3^o$, So Area = $5.33\ U^2$.

How did I find <A? The Generalized Pythagorean Theorem of course, aka Law of Cosines. For five videos which thoroughly explain this, visit:

www.*TriadMathInc*.com/m19

Chapter 6

The High School Math Ed Delivery System.

Revamping our High School Mathematics Curriculum must start with our Delivery Systems. Modern technologies empower us to create and deliver a much different mathematics education to our students than their parents received.

This difference is just as profound as the many other technologies affecting and directing their lives, like Cell Phones, Google, Facebook, Kindle, I-pads, Games, etc..

In the United States our current high school curriculum and delivery system has roots going back over a century. It is very expensive and inefficient for many obvious reasons.

Lectures and group synchronization are simply not very effective in teaching mathematics. Indeed they are often counterproductive.

Today we have modern communications technologies that empower us to deliver education in a much more efficient and cost effective way. And, better yet, since it can be individualized and self-paced it is much more effective for the student pedagogically.

Failure to dramatically improve our mathematics education both qualitatively and quantitatively will result in much less opportunity for our student graduates.

TIME is of the essence. Today's system wastes immense amounts of our students' and teachers' time. Time wasted is literally money wasted. "Free education" is not free if it wastes time.

Consider the following thought experiment. What is a student's time worth? Say, a minimum of $5 per hour. A teacher's time? Say, $30 per hour.

Suppose we use some "free" materials or resources and achieve a certain result with ten hours of the teacher's time and 200 hours of students' time (20 students times 10 hours each).

This costs a total of $300 + $1,000 = $1,300. We note the students' time is always worth more in aggregate than the teacher's time.

Now suppose we could obtain some materials for $200 that reduced both the teacher's and student's time by 50%. This would result in a $650 savings in time for only $200. A pretty good ROI.

Of course, the teacher's time savings was only $150 so it might appear that our costs have increased by $50 if we ignore the students' time value.

We often ignore the value of our students' time and this is not only a great disservice to them, it sends the wrong message.

Today, one can obtain excellent resources online for a very small fraction of what it would cost to produce them on site even in a group setting. Students can study material at their own pace on their own schedule and get as much help as they need online. They also can help each other and this is a great way to learn, by 'teaching' a peer.

One example of this is the Dr. Del Ten Tier Program. In time, there will be more and more variety and specialty educational programs.

Most importantly, it is imperative that our students learn to use the modern tools and technologies as they learn the concepts.

More than ever in our modern economy life long learning will be mandatory for most workers as our technologies evolve and we all become more and more productive as the tools evolve and improve.

It is incumbent on our educational leaders and parents to research and utilize the most effective methods and content available.

Of course, method does not replace content.

In mathematics, content can be a matter of opinion and judgment. Obviously, I have some strong opinions about what are appropriate materials.

Ultimately each parent or educator or responsible party will have to make such judgments. For the sake of the students make a good one.

To get an idea of some of the resources that are available today and their costs you may visit:

www.TriadMathInc.com

and look at the various "products" available and their costs. *As you will realize you can deliver high quality training to your student for a few dollars per month, much less than the cost of a live tutor*.

My advice to you is to always consider your student's time in any resources you use.

Also, be sure the resources you choose are meeting the Effective Math Education Triad's principles. Otherwise, you may just be wasting your money and your student's opportunities to learn.

Chapter 7

Financial Salvation for our High Schools

We live in perilous economic times. Every business and governmental institution is under financial pressure. Education is no exception. We must find innovative solutions if we are to simultaneously improve our students' education and reduce our costs.

There is a way!

If our ancestors wanted to hear some music they had to create it themselves or engage some local musician to create some. Can you imagine the quality of music available to most of our ancestors?

Sure, in a few very rich cities a few very rich citizens could employ the best musicians of the day to create and perform music. But, prior to the 20th century most of our ancestors never got to hear a great musician perform, and never got to hear a Mozart sonata or a Beethoven symphony.

Thanks to modern technologies starting with the Victrola (record player) and radio many people got to hear great music from the best musicians of their era.

Today, we take it for granted that the best music that man can create is available to virtually anyone at a very low price point. Great music for everyone! A similar situation applies to books and plays.

But, education is still delivered the way music was a hundred years ago. It is essentially created and delivered locally. And, truthfully, it is often of very poor quality.

Mathematics is very difficult to teach effectively. First, one has to understand math at a deep level, and that is virtually impossible for most middle and high school teachers, through no fault of theirs.

The textbooks try to mitigate this, but they are pretty ineffective, and actually can impede a good teacher. Most students can not learn from a book on their own.

The truth is we need to drastically revamp our educational delivery system.

Online delivery of education is the future.

You can get experts to create content that can then be delivered in a self-paced and interactive manner to the students, with proper exercises and quizzes for practice and feedback. *The most cost effective way is with an automated delivery system.*

Do teachers still have a role to play?

Absolutely! Better than ever.

Teachers will become more like mentors or coaches, and help the students properly utilize the system. A teacher will be free to help the students who need special help unavailable from the automated delivery system.

Today, some schools offer traditional courses using an online delivery method. It is somewhat more efficient than a classroom method, but still requires an expert teacher, which is very difficult to come by in many cases.

Remember, Content is an overwhelming component of the Triad. And, usually this is only available from an expert.

In the United States, we are faced with archaic, inefficient, and ineffective methods and delivery systems supported by a vast entrenched educational system.

When technology changes things, there are perceived losers. But perception is not always reality. Clearly the roles of teachers will dramatically change in the future.

Perhaps counter-intuitively, teachers will be more empowered to succeed along with their students.

It is imperative that teachers learn how to be proactive and to help their students with these new innovative educational modalities.

The opportunities for meaningful one on one interaction with both the weakest and strongest students will increase, and special projects and class discussions, and on and on. The possibilities are endless. Teachers may benefit as much as the students.

Now, let's talk money.

Most of our high schools today are faced with financial struggles and declining budgets. Let's just talk about the mathematics programs.

Suppose you could deliver a great math education to your students that cost about one half as much as your current program.

This savings could then be applied to other areas where perhaps technology couldn't be as useful or effective, or ease your budget woes.

Suppose your students did much better. Suppose more of them learned more math and, thus, were able to pursue more lucrative careers.

On a wide scale we can achieve these financial goals. I think it probably could be achieved in several fields besides math too. Probably various science subjects and some social science subjects could be taught in this way.

The schools would become learning centers whereby the students could be provided the necessary technology

and wise mentors to use this technology to obtain a much better education than is being achieved today.

Quality and Productivity UP, Total Costs DOWN.

Teachers will also be more empowered to be more effective by helping and leading students in many ways.

One-on-one mentoring - - - discussion groups - - - supplementary materials - - - special projects - - - and many other things will now be available to teachers and students once more automated materials are utilized.

"Textbooks" should be replaced with effective multimedia automatically delivered, interactive and self-paced materials created by experts.

Teaching math can much more fulfilling and enjoyable. It will be like having an army of expert tutors to help with each student according to the student's needs.

One word of warning. The media will not substitute for the content. In fact, in some cases the media may detract from the content. This will have to be evaluated very carefully. The only criteria that should ultimately be used is how effective the education is for the student.

Interlude #4 Archimedes Tombstone

Archimedes was the greatest Greek mathematician of antiquity. He made many profound discoveries, including that the area of a circle is (C/2)x(D/2) [= πR^2] which is quite amazing in itself. Remember $\pi = C/D$ by definition.

However, Archimedes made a more profound discovery he wanted inscribed on his tombstone. We use it in the Foundation as a memory device to remember the formulas for the Volume and Surface Area of a Sphere. We prove it in Tier 3 following Simmons book.

Archimedes discovery was that both the Volume and Surface Area of a Sphere are exactly 2/3 of the Volume and Surface Area of a Cylinder that circumscribes the Sphere. Since it is very easy to derive the formula for the Volume and Surface Area of the Cylinder, it is then very easy to derive the formulas for the Sphere.

We present this in the Geometry portion of the Foundation for several reasons.

1. It shows the relationship between two different objects, a cylinder and sphere.
2. It is a good memory device for remembering the formulas for the sphere.
3. It gives the student a sense of history and the profundity of what our ancestors went through.
4. It is fun.

Visit: www.*TriadMathInc*.com/m20 for more details.

Chapter 8

The "Matheracy" Imperative

Our future will be so different from our past and present that we may need a new word, "matheracy". The word isn't so important. The underlying imperatives and concepts are extremely important if we are to prosper and maintain any type of leadership in the world.

It is obvious to all that we live in a new rapidly changing economic environment. Thanks to modern transportation and communications technologies we all compete with each other on a world wide basis.

Life long learning is mandatory for a successful career in virtually any field. Machines and humans are becoming more and more integrated. By "machines" I include computers and software.

The synergies between machines and humans are creating incredible productivity increases in many areas. So, this gives humans unprecedented opportunities for growth in material productivity and wealth.

We have a precedent for this to learn from, the so called "agriculture revolution", that Alvin Toffler described in his great 1980 book, *The Third Wave*. The First Wave of civilization was the Agriculture wave.

Most humans gravitated from a pre-civilization hunter-gatherer economy to the First Wave of civilization, based on basic First Wave agriculture technologies. They became more "prosperous" and, indeed, that is when both technology and populations "exploded".

Then, came the Second Wave of modern civilization based on mechanical, hydraulic, electrical, steam, oil and Newtonian/Maxwellian technologies, much more sophisticated and powerful technologies, 1750 - 1950.

Starting in the mid 1800's, the agriculture economy was revolutionized which resulted in much more productivity and much less employment.

Today, agriculture is basically carried out in this Second Wave economy by modern technologies and agriculture workers that are highly skilled and educated and married to machines which are the key to their productivity. More agriculture products are produced with fewer workers. Productivity is "way up"

In the Second Wave, manufacturing became the new driving economic force and employed a majority of humans. Many of our current social institutions and political structures were created to accommodate this Second Wave economy.

Then, with the advent of modern electronics technologies (Quantum/Einsteinian) which include computers and communications technologies, we entered

the Third Wave of civilization. These new technologies are orders of magnitude more powerful than the Second Wave technologies.

In this Third Wave, manufacturing became more and more productive and the number workers in manufacturing declined as they became more and more educated and productive. Déjà vu.

In the Third Wave economy, material goods will become more and more plentiful and inexpensive. More and more people will work in the so-called service economy.

For some people, this “service economy” conjures up the image of “burger flipping” and low paid entry level jobs. *This is a very poor and inaccurate conception of the Third Wave service economy.* In fact, it can be very misleading to the naïve.

Service jobs will increasingly involve utilizing the “machines” of modern technology. Think of a highly trained doctor or electrician or construction engineer as the model for what we call “service jobs”. Now multiply by thousands of job types.

In fact, workers with a high degree of ability to utilize modern technologies will be better and better compensated. The modern Third Wave “middle class” will have much more material comforts than the middle class

of the Second Wave, just as they prospered compared to the middle class (peasants) of the First Wave.

This means that in our modern Third Wave economy workers with a high level of education and modern skills will be in a wonderful position to prosper. **$100,000 to $500,000 per year incomes will be common and wide spread for properly educated Third Wave workers.**

The flip side of this coin is that workers who are not educated or skilled will have a relatively poorer position, even though it will be pretty good compared to so-called poor people of today.

In fact, one can see an analogy today by looking at societies and economies that are not so technically advanced. There are people today in this world who are still working in agriculture in a First Wave mode. These people are materially extremely poor compared to workers in modern economies. It is reported that some of these are the people living on a dollar or two per day.

So what can or should we do? There is only one answer really. ***Give our students an appropriate education to propel them into successful careers and jobs in this new economy.***

Language skills and human interaction skills are important just as they were in the Second Wave. Literacy was a great Second Wave achievement.

Now hear this, mathematics is more important in this Third Wave economy than it was in the Second Wave economy.

We owe it to our children and students to give them a great mathematics foundation since mathematics knowledge and skills will help them learn and interface with the “machines” of our modern technologies.

It is imperative we deliver our students a great mathematics education. We are not doing this today in the U.S. as many reports and studies are revealing. We must change this ASAP. That is the purpose of this book, and Dr. Del’s “crusade”.

The Mission of Triad Math is to help math teachers everywhere rectify this situation by providing tools, materials, and leadership to help them deliver a modern first rate math education to their students.

The future is very bright for highly skilled and knowledgeable people. Semi-skilled workers in the Second Wave could “get away” with mathematical ignorance and incompetence. But, this is a rapidly shrinking class.

Many Third Wave workers in the “middle class” will have to be technically skilled, and this often will require mathematical knowledge and competency.

Just as literacy was a requirement for success in the Second Wave, mathematical competency will be mandatory for success in the Third Wave.

There is not even a word for this so far as I know. "Numeracy" has been used. But, mathematics is much more than just numbers. Maybe a word like "matheracy" will have to be invented.

Beyond doubt, if we want our students to be successful as they compete with workers from all around the world, then it is mandatory they be properly and adequately educated in mathematics. Not only must they be literate, they must be matherate.

This is not happening in the United States today, by a long shot. But, it can be corrected immediately if we simply utilize our existing technologies and revamp our math education system. That is what Triad Math, Inc., and the Dr. Del's Ten Tier Program is dedicated to.

Surely, the proper mathematics education of your child or student is more important than a new cell phone or video game or designer cloths.

It costs no more today.

Thus, we have NO EXCUSES.

Chapter 9

The Ultimate Benefits to our Economy and Society

Benefits. Benefits. Benefits.

Isn't that what we all want in one form or other? Material benefits are often prerequisite to spiritual and emotional benefits. Educating our children is one of the benefits we all want for ourselves and our children. This "crusade" is all about achieving this benefit.

We live a very integrated interconnected world thanks to modern technologies. We are totally dependent on the complex economy and systems that have evolved.

How long do you think you could survive if all electricity was cut off for some reason? What would society evolve into in such an event? Ugh. Horrible to contemplate isn't it?

If one thinks just a little about all the things we depend on for our existence and comforts, it soon boggles the imagination.

We are about seven thousand million people (7,000,000,000, seven billion) spread over the earth. When there is a disaster of some type and 100,000 people die, statistically this is only 1 in 70,000 humans dead.

Of course, if you or a loved one is one of them it is a very overwhelming disastrous event. But, every day thousands of human beings die without affecting us in any personal way.

On the other hand, if there is a natural disaster that destroys some resource that is critical in our complex network then that can affect many more people.

Losing a valuable trained human resource that can contribute to the economic society is also a loss felt by everyone. The failure to properly educate a person adequately so s/he can be a valuable contributor is a loss to economic society, and is such an unnecessary loss.

A little "thought experiment". Imagine a society with 1,000 people in it. 100 are productive thanks to their education, skills, and ambitions. 900 are non-productive. If the productive people produce 50 units of products per person there will be 5,000 units to be shared in some way by 1,000 people, or <u>5 units per person.</u>

Now imagine a society with 1,000 people with 200 productive people. Then there would be 10,000 of units to be shared or <u>10 units per person.</u>

Hmm. Twice the material goods to share. Which do you think would be the better society economically? And, socially? Scale this by a million to see the effects on modern economies.

Now, I am not talking about capitalism versus socialism, or any other economic system. That's a topic for another discussion I'll leave to P.J. O'Rourke.

I am just pointing out the obvious fact that the higher the percentage of our population that is productive; the better we are all off economically, and many other derivative ways.

The key to productivity in our modern Third Wave economy, with its technology and "machine amplification" of human skills, is education. And, mathematics is the foundation of most quantitative and technical skills.

So, the better we educate those inclined to this type of job or career the better off we will all be.

The good news is that with modern technologies we can do a terrific job of giving a quality math education to virtually all students who would like to benefit from it. Ironically, it also can be much less expensive than our current dysfunctional math education system.

One of the keys is to be sure the student's psychology is right regarding math and then teaching them math in a tiered or layered approach with feedback and self pacing.

This can be done today quite economically, especially compared to our old antiquated Second Wave system of delivering math education in a "factory" mode.

For only a few dollars per student per month we can deliver very high quality math education which can in turn serve as a foundation for myriad career paths or jobs.

For less than the cost of an expensive set of textbooks, we can deliver excellent multimedia materials online that are vastly more effective than any textbook, and much more empowering to both our students and teachers. The Return on Investment to our overall economy will be staggering. Just compare the effects of a non-producer to a producer.

Compare Scenario 1 above to Scenario 2.

Societies who take advantage of modern technologies to educate their citizens will prosper greatly in comparison to those that don't.

Because the U.S. was one of the leaders of the Second Wave, its obsolete Second Wave educational systems are thoroughly entrenched. It will be a major challenge for the advanced industrial Second Wave societies to evolve into a Third Wave system.

Fortunately, thanks to the freedoms the U.S. enjoys, a grass roots awareness and determination of individual parents can motivate and lead to this transformation.

Indeed, could this be led by the rapidly growing "home school" industry? Or, perhaps by Charter schools, or Private Schools, or innovative Public Schools?

Ultimately, it will be necessary for the U.S. public education system to adapt to these new realities, if the U.S. is to maintain its leadership position in the world.

Thanks to the power of exponential growth this could happen very fast. Public high schools are in somewhat of a financial crisis today for a variety of reasons that aren't going away soon. Adopting new education delivery methods along with much better content at a much lower cost may be the answer to this problem. Win – Win - Win

Indeed, high schools that adopt these new Third Wave approaches to education will thrive and lead the way. This will apply to either public or private or charter schools.

Make no mistake about it. The future of our economy and the relative prosperity of our citizens depends on our educational system. More than ever, each and every child needs to be offered a "world class" education. Not because of some "do-gooder" moral imperative, but because of hard-nosed economic reality.

A child who is given a great education will be productive and that will benefit all of us, compared to a child without an education who we must then support. The child with a good education will be much happier than the child we attempt to support.

The more happier economically successful citizens we have, the better our society will be. Scenario 2 societies are usually more satisfying than Scenario 1 societies.

I believe it is much better to raise everyone's economic well being, and this is achievable today if we apply modern Third Wave technologies to our educational system.

You are invited to join us in this "crusade". Let's be sure any students you are involved with are getting a first rate high school mathematics education for all the reasons enumerated above. There is absolutely no excuse for this to not happen today.

Any parent of a student who is struggling with math, can now obtain this type of education for their child for a few dollars per month, a small fraction of what a private tutor would cost. There is no excuse today for any student to be left behind.

Just visit: www.TriadMathInc.com for information.

Interlude #5 Complex Numbers and Trigonometry

The Complex Number system is important for many reasons. Here is just a small taste of it for math teachers.

The Real Numbers correspond to points on a straight line. This we teach in high school today.

Complex Numbers correspond to points in a plane. I will assume you understand the Complex Numbers the way we teach them in Tier 4. So you know:

Two complex numbers **X** and **Y** of length **1**, on the unit circle, with angles **<A** and **<B** from the real axis are:

$$\mathbf{X = COS(<A) + iSIN(<A)}$$

$$\mathbf{Y = COS(<B) + iSIN(<B)}$$

What I consider the most important equation in math, for multiplication **x** of complex numbers, says:

Geometric Def of **x** = Algebraic Def of **x**. Or:

$$\mathbf{COS(<A + <B) + iSIN(<A +<B) =}$$

$$\mathbf{[COS(<A) + iSIN(<A)] \ x \ [COS(<B) + iSIN(<B)]}$$

From this it follows:

COS(<A+ <B) = COS(<A)COS(<B) – SIN(<A)SIN(<B)

SIN(<A +<B) = SIN(<A)COS(<B) + SIN(<B)COS(<A)

That's how I remember the formulas for these trigonometric identities plus many more in a similar manner.

Trigonometry and Complex Numbers are intimately related. In fact, it may be the best way to really understand Trigonometry, which we utilize in Tier 4.

Complex Numbers are required to "solve" algebraic equations, starting with the quadratic equation.

Visit: www.*TriadMathInc*.com/m21 for more info.

Interlude #6 will discuss an even more profound relationship involving complex numbers.

CODA

Chapter 10

Short Changing our Best Math Students

What is more important to us as parents and educators than delivering an adequate education to our students? Very few things I would say.

Until this 21st Century, the best we could do is what we are doing today. Batch teaching in our massive school "factories". A Second Wave approach.

Today, our modern technologies empower us to deliver a Third Wave type of education that is vastly superior in virtually all ways to our current system, . . . and more cost effective too.

But, what this means for our brightest and best math students is even more dramatic.

This is the "icing on the cake" for this crusade. And, it may one of the most important things we can do for the long term health of our society..

Imagine that we did not have youth sports activities, both in and out of our school systems. No Little League, or Junior Football.

Suppose our athletes had to wait until their late teen years to obtain good sports education and coaching. How good would our best athletes be?

Malcolm Gladwell discusses these issues in his book *Outliers*. See Malcolm's 10,000 Hours discussion from the Beatles to Bill Gates.

Beyond any reasonable doubt early education and practice in any skill and knowledge based area is critical and virtually mandatory for future experts.

Learning is cumulatively exponential in nature. This means that a person who starts one year earlier than his or her peers has a distinct advantage, all other things being equal. The same thing applies to music and other arts, and mathematics.

For most people, it won't matter too much. If you are only going to be an amateur athlete or musician it doesn't matter too much when you start. Same for mathematics.

But, for future experts it does matter very much.

Mathematics is a huge subject. There is as much mathematics as there is music. Just imagine how much music has been created and is continuously being created. The same applies to mathematics, believe it or not.

Now, for people who are just going to apply mathematics to some quantitative subject like engineering or economics, not too much math is required.

Through Tier 6 in high school would be more than adequate for most students. It would be outstanding.

But, it's just a drop in the bucket to a mathematician. To be a future professional mathematician much more will be required just like it requires much more to be a professional athlete or musician.

We are drastically shortchanging our best math students with our current system.

Until recently, this was unavoidable due to lack of expert resources in our schools. This was understandable and justifiable due to financial and logistical constraints.

But, today things are different. It is now possible to deliver excellent math education to any student anytime anywhere thanks to modern technologies.

In the last Chapter 11, I will discuss various topics we should be educating our best math students on very early while they are still in their early and mid teen years.

The Plan for the Dr. Del Ten Tier program is a resource one can use as a model to see what can and should be done. This program can easily be utilized by the more advanced precocious students who think they might wish to become mathematicians.

I plan to elaborate on this in a sequel to this book.

Sequel Chapters (tentative)

1. The Axiomatic Method
2. Math Models
3. Modern Math Structures
4. Chaos and Non-linear Dynamics
5. Spectral Logic
6. Fractals and non-integral dimension
7. Infinity
8. Axiom of Choice
9. Infinitesimals
10. Impact on science
11. The Black Swan and risk analysis
12. Other topics TBA

Of course, "Man Plans and God Laughs" is my favorite Mantra. So who knows if this Sequel will ever appear? It might depend on you, and your reaction to this book. I'm no different than you or anyone. I need "support" and fuel for motivation and drive.

I will conclude this book in Chapter 11 with a description of Tiers 7 thru 10 for those students who contemplate a career in mathematics.

Interlude #6 $i^i = ?$ where $i = \sqrt{-1}$

Understanding the Complex Number System is very easy IF it is presented properly. I use this as a quick "test" to see if a person understands it properly.

As you may know, our ancestors had a very difficult time, spanning centuries, getting it right. If presented improperly it is not easy to understand complex numbers. In Tier 4 we present it the way we find it easiest to understand.

There is a strong correlation between the geometry and the algebra of Complex Numbers. This is the KEY to understanding them. The complex numbers on the Unit Circle are expressed by:

$$e^{i\theta} = \mathbf{COS}(\theta) + i\mathbf{SIN}(\theta)$$

There is an easy way to explain and justify this notation we explain in Tier 4. Thus,

$$i = e^{(\pi/2)i}\text{, So, } i^i = [e^{(\pi/2)i}]^i = e^{-(\pi/2)} = .2079$$

Kind of a curiosity, but tests your understanding of Complex Numbers.

For more; www.*TriadMathInc*.com/m22

Chapter 11

Dr. Del's Tiers 7 thru 10, and more.

This is for those parents or teachers who might have a precocious math student who might wish to consider a career in mathematics. This will not be an easy chapter for people not well educated in mathematics. But, it will give a layman an idea of what is needed for those relatively few precocious students who are potential mathematicians.

Tiers 1 – 2 prepare non-professional (vocational) students for a career in any quantitative technical field. This is a great way to earn a really good living in our modern economy. This Practical Math Foundation is a foundation for all that follows.

Tiers 4 – 6 prepare a student wishing to pursue a STEM career and matriculate to any university or college. STEM = Science, Technology, Engineering, and Math.

What about students with a high aptitude for math and considering a professional mathematics career?

No high school I am aware of has a program that is adequate for these students. Most of them will go on to college and major in math and, if it's a first rate university, learn things they will need to know, - - -years too late.

Imagine starting a professional athlete's or musician's advanced training only when they enter university. We know from experience this is a recipe for disappointment and failure. Very late starters can rarely catch up with their better educated and prepared peers.

Mathematicians are no different from athletes or musicians. They need advanced sophisticated coaching and training as early as possible.

Tiers 7 thru 10, when completed, will present things that mathematicians should learn at a very young age, as soon as they are ready for it.

Here is a description of the Plans for Dr. Del's Tiers 7 thru 10 for the precocious math student. A nonmathematician will probably have difficulty understanding this discussion. Many of these topics may be discussed in a little more depth in their own Chapters in the Sequel.

Tier 7

Advanced theoretical math will be discussed starting with a theoretical description of the various number systems, the axiomatic approach, math models, etc. Logical inference, fuzzy logic, Boolean algebra, proof techniques and other topics foundational to further theoretical math studies will be covered.

There are many topics that will be discussed in preparation for the last three Tiers.

The emphasis will be on the beginning of the development of that elusive quality called "mathematical maturity" math professors discuss.

Tier 8

Some of the most important basic theoretical mathematical techniques will be introduced. Building new math structures from existing math structures using such techniques as cross products and then quotient structures is a critical skill.

For example: Building the Integers from the Natural Numbers; then the Rational Numbers from the Integers; then the Real Numbers from the Rational Numbers; and then the Hyperreals from the Reals using ultrafilters. All of this was developed in the late 19^{th} century except the last one, which was developed in the 1960's.

Quotient structures are often a major conceptual hurdle aspiring mathematicians must cross. Once this concept is mastered the student will be ready for more advanced theoretical math courses at a good university.

A major increase of that elusive quality mathematicians call "math maturity" will be developed here.

Tier 9

More advanced abstract math will be discussed. Many of the math structures used by mathematicians will be introduced, such as:

Topological Spaces
Groups
Rings
Fields
Linear (Vector) Spaces
Hilbert Spaces
Measure Theory
Modern Analysis
Modern Algebra

and more, will all be discussed from an overview conceptual point of view. Chaos, Non-linear systems, Catastrophe Theory will be discussed.

The purpose of Tier 9 is to give the student an overview understanding of modern math structures and fields to help determine what directions he or she might wish to pursue and to open his/her mind to the possibilities.

Oh, how I wish I could have had such an introduction to modern math before I embarked on my graduate studies.

Most people are totally unaware of what modern math consists of. It is HUGE, beautiful, and powerful. It is the basis of our modern civilization's technologies.

Tier 10

This could come before or in place of Tier 9. Concepts of Infinity, Godel's Theorem will be discussed along with Russel's Paradox, Cantor's methods, The Axiom of Choice and Zorn's Lemma and Spectral Logic.

Model theory and the fascinating story of infinitesimals will be told. It's an amazing story spanning three millennia from Archimedes to Abraham Robinson, with profound implications for the future.

How math models apply to science. The pendulum as a cautionary tale of how an over simplistic linear model can hide or overlook deep behaviors (Chaos) will be discussed.

Some brief discussions of some math models used in science such as those used in Quantum Theory and Relativity Theory and Lie Groups to the Standard Model of modern physics.

By the end of Tier 10, a student should have a pretty good idea about what mathematics is all about and how much more they might wish to learn.

The goal of these last four Tiers is to launch an aspiring mathematician properly into his or her career, and to be sure they know what they are getting into.

Dr. Del learned much of this after he got his Ph.D. He feels it would have been invaluable to have learned it many years earlier than he did.

Because math knowledge and maturity grow exponentially, it is imperative one start as early as possible if one is to have a successful career. You wouldn't want to wait until you were in your twenties to begin to seriously learn music if you wanted a career in music. The same applies to math.

Ten Tiers Timeline for a precocious student

The last four Tiers, 7, 8, 9, and 10, are for those students who are contemplating a career in mathematics.

The timeline for the Ten Tier system will depend on the particular student. Each Tier is designed to be taken in a self-paced manner. See page 59 for the first six Tiers

It is important that each student absorb and understand the material in a way comfortable for him or her. What follows is what could be a typical timeline for a precocious student.

If a student gets through Tier 6 by high school graduation he or she will be far ahead of his or her contemporaries from typical schools, both public and private.

However, a precocious student with high aptitude for math and a strong work ethic may finish the first six Tiers long before high school graduation.

Most high schools have little to offer them. Here is what we believe they should be offered.

Tier 7 is projected to take another three to six months and will advance a student's understanding of the theoretical basis of the things they have learned in the first six Tiers.

Tier 8 is also projected to take another three to six months. This will be very dependent on each particular student.

We estimate that Tiers 7 and 8 will consume about one year of time for many students who proceed this far. *Then, for students who are really want to understand what higher math is all about, Tiers 9 and 10 will give them an amazing insight and overview of modern mathematics.*

Tier 9 is also projected to take another three to six months. This, too, will be very dependent on each individual student.

Tier 10 will be very similar to Tier 9 in time required.

So, to summarize. The last four Tiers could take many precocious students about two years.

It is quite possible to cover many of these concepts fairly quickly from an overview point of view. Remember that the student's "mathematical maturity" will also be developing, and this is just as important as the new concepts they are learning.

Most students of math don't get exposed to these concepts until they are quite advanced in their studies. But, it would be of great value to learn about these things earlier in one's studies. It takes time to absorb and digest these ideas and the sooner one starts the better.

Here is a possible time line for a precocious student.

7^{th} Grade Tiers 1 and 2 and 3
8^{th} Grade Tiers 3 and 4
9^{th} Grade Tiers 4 and 5
10^{th} Grade Tiers 5 and 6
11^{th} Grade Tiers 7 and 8
12^{th} Grade Tiers 9 and 10

You should notice the "overlap". Math is learned in a non-linear way. One often must "loop back" to previous material in order to deepen their understanding. Usually, one gains deeper understanding and insight when one "reviews" the material.

Call to Action

If you are involved in any way with a high school student who is struggling with math or who wants to learn math please have him or her study the materials at:

www.TriadMathInc.com

Then, if and when they are ready they can start with the Foundation Course. ANY parent can do this. You don't have to be a teacher. This will apply to any person who just wants to get a good foundation in practical math for a future job or career in some technical field.

This also applies to any student who aspires to a career in some STEM subject. The Foundation is designed to also provide a foundation for future math studies as outlined in this book.

If you are a teacher or mentor of math, then you should visit: www.TriadMathInc.com and download the PDF free version of Math Teachers Guide and join the Dr. Del Math Teacher's Club, open to all and free too.

Finally, you may contact me via email at: drdel@TriadMathInc.com or Phone: 812-355-3030

Special Message To:

Homeschool Teachers of a High School Student

There is a way to . . .

"Give Your Child and Excellent High School Math Education Regardless of Your Own Math Ability."

Here's how.

<u>First</u>, complete the Dr. Del Practical Mathematics Foundation Course.

This is an:

- **Interactive**
- **Self-paced**
- **On-line**

Practical Math Foundations Course delivered online via the Internet, which will teach the <u>appropriate topics</u> from:

- **How to use the TI 30XA scientific calculator**
- **Pre-algebra**
- **Algebra**
- **Geometry**
- **Trigonometry**

your student will need to know to solve most practical everyday problems that arise in most technical subjects, with No Fluff.

How this is accomplished is fully explained at our website:

http://TriadMathInc.com/the-foundation-online-course/

Most students take about fifty hours of self paced time to complete this Foundations course.

When done your student will know more math than 95% of U.S. adults.

Then, continue with the upper Tiers when they become available until your child has received the best math education possible.

The upper Tiers are planned to go through Calculus and far beyond for students who anticipate a career in mathematics.

The upper Tiers will also be self paced and interactive.

In both the Foundation Course and the upper Tiers, the student is given a Notebook with notes on each topic which accompany the video lesson. Indeed, Tiers 3 and 4 also will utilize the wonderful book by Dr. George Simmons, *PreCalculus Mathematics in a Nutshell*.

The student may watch the videos in an interactive and self-paced way on his or her own schedule.

Then, there are targeted Exercises in the Notebook, with complete answers, which will lead to complete mastery of the topic.

Then the student may take a short online Quiz enabling the student to demonstrate his or her understanding of the topic covered.

Working independently a typical student will be able to complete as much mathematics as they desire.

This program is not achievable easily in a standard classroom for reasons fully discussed in Dr. Del's book *Teaching Math*.

"What will this cost?" is the typical next question.

"What would it be worth for a student to be able receive an outstanding mathematics education" is a question you should answer first.

Pricing information will be available at the Website:

www.TriadMathInc.com/order

It may change from time to time.

However, you probably will be surprised by how cost effective it is, especially compared to any alternative that would produce such a good outcome for your student.

For additional information you may contact:

Debbie Goodman at: 812-355-3030 or email:

dgoodman@TriadMathInc.com

Special Message To:

Parents and Teachers of a High School Student

There is . . .

"A Way to Pass the ECA Math Exam, and Provide an Excellent Foundation for Future Math Studies, in Less Than One Semester"

Here's how.

<u>First</u>, complete the Dr. Del Practical Mathematics Foundation Course.

This is an:

- **Interactive**
- **Self-paced**
- **On-line**

Practical Math Foundations Course delivered online via the Internet, which will teach the <u>appropriate topics</u> from:

- **How to use the TI 30XA scientific calculator**
- **Pre-algebra**
- **Algebra**
- **Geometry**
- **Trigonometry**

your student will need to know to solve most practical everyday problems that arise in most technical subjects, with No Fluff.

How this is accomplished is fully explained at our website:

http://TriadMathInc.com/the-foundation-online-course/

Most students take about fifty hours of self paced time to complete this Foundations course.

When done you will know more math than 95% of U.S. adults.

Second, complete the ECA extension course.

There are certain topics needed to excel in the ECA exam that are not covered in the Practical Math Foundation Course.

This ECA extension should take you another thirty hours or so.

It, too, is self paced and interactive.

In both the Foundation Course and the ECA extension, the student is given a Notebook with notes on each topic which accompany the video lesson.

The student may watch the video in an interactive and self-paced way on his or her own schedule.

Then, there are targeted Exercises in the Notebook, with complete answers, which will lead to complete mastery of the topic.

Then the student may take a short online Quiz enabling the student to demonstrate his or her understanding of the topic covered.

Working independently a typical student is able to complete both the Foundation Course and the ECA extension in about one semester if s/he works one hour per day five days per week, on average.

This program is not achievable easily in a standard classroom for reasons fully discussed in Dr. Del's book *Teaching Math*.

"What will this cost?" is the typical next question.

"What would it be worth for a student to be able to pass the ECA Exam and be fully prepared for future math studies?" is a question you should answer first.

Pricing information will be available at the Website:

www.TriadMathInc.com/order

It may change from time to time, but you will probably be surprised by how cost effective it is.

For additional information you may contact:

Debbie Goodman at: 812-355-3030 or email:

dgoodman@TriadMathInc.com

Special Message To:

Future Tech School Student or Adult Learner

There is . . .

"Do well on the COMPASS Exam in Less Than One Semester for less than $100"

Here's how.

<u>First</u>, complete the Dr. Del Practical Mathematics Foundation Course.

This is an:

- **Interactive**
- **Self-paced**
- **On-line**

Practical Math Foundations Course delivered online via the Internet, which will teach you the <u>appropriate topics</u> from:

- **How to use the TI 30XA scientific calculator**
- **Pre-algebra**
- **Algebra**
- **Geometry**
- **Trigonometry**

that you will need to know to solve most practical everyday problems that arise in most technical subjects, with No Fluff.

How this is accomplished is fully explained at our website:

http://triadmathinc.com/the-foundation-online-course/

Most students take about fifty hours of self paced time to complete this Foundations course.

When done you will know more math than 95% of U.S. adults.

Second, complete the COMPASS extension course.

There are certain topics needed to excel in the COMPASS exam that are not covered in the Practical Math Foundation Course.

This COMPASS extension should take you another thirty hours or so.

It, too, is self paced and interactive.

In both the Foundation Course and the COMPASS extension, the student is given a Notebook with notes on each topic which accompany the video lesson.

The student may watch the video in an interactive and self-paced way on his or her own schedule.

Then, there are targeted Exercises in the Notebook, with complete answers, which will lead to complete mastery of the topic.

Then you may take a short online Quiz so that you can demonstrate your understanding of the topic covered.

So, working independently a typical student is able to complete both the Foundation Course and the COMPASS extension in about one semester if s/he works one hour per day five days per week, on average.

This program is not achievable easily in a standard classroom for reasons fully discussed in Dr. Del's book *Teaching Math*.

"What will this cost?" is the typical next question.

"What would it be worth for a student to be able to be fully prepared for entry into tech school?" is a question you should answer first.

Pricing information will be available at the Website:

www.TriadMathInc.com/order

It may change from time to time. However, you will probably be pleasantly surprised by how cost effectively it is priced, and how much of your time will be required.

For additional information you may contact:

Debbie Goodman at: 812-355-3030 or email:

dgoodman@TriadMathInc.com

Special Message To:

High School Decision Makers

Principal, Superintendant, Board Member

You have some students who are not thriving in your standard math curriculum for some reason. It's probably not anyone's fault, but it is hurting both the students and your school.

Thanks to modern technologies there is now something you can do to rectify this situation very cost effectively.

There is a way to convert some of these "failing" or struggling students into math "successes".

It can be done for less than $100 per student.

It does not involve any math teacher and it does not require any changes to your current math curriculum or program.

This transformation can be achieved with an Online Program described in this Message. This program will result in the student becoming "matherate" in the sense of

learning all of the practical mathematics needed in most everyday technical fields.

It will result in the student being able to pass the ECA exam.

It will prepare the students for further math studies in the event they then choose to continue pursuing a STEM career path.

Quality and Productivity UP

Total Costs DOWN.

Here's how this can be achieved.

First, complete the Dr. Del Practical Mathematics Foundation Course which is:

- **Interactive**
- **Self-paced**
- **On-line**

Practical Math Foundations Course delivered online via the Internet, which will teach your students the appropriate topics from:

- **How to use the TI 30XA scientific calculator**
- **Pre-algebra**
- **Algebra**
- **Geometry**
- **Trigonometry**

your students will need to know to solve most practical everyday problems that arise in most technical subjects, with No Fluff.

How this is accomplished is fully explained at our website:

www.TriadMathInc.com/

Most students take about fifty hours of self paced time to complete this Foundations course.

This course does not require a math teacher. It would be good for the student's to have a mentor, or leader, to be sure they are properly participating.

When the student completes the Foundation Course the student will know more math than 95% of U.S. adults and be ready to embark on a non-professional technical career.

Second, complete the ECA extension course.

There are certain topics needed to excel in the ECA exam that are not covered in the Practical Math Foundation Course.

This ECA extension should take you another thirty hours or so. It, too, is self paced and interactive.

In both the Foundation Course and the ECA extension, the student is given a Notebook with notes on each topic which accompany the video lesson.

The student may watch the video in an interactive and self-paced way on his or her own schedule.

Then, there are targeted Exercises in the Notebook, with complete answers, which will lead to complete mastery of the topic.

Then the student may take a short online Quiz enabling the student to demonstrate his or her understanding of the topic covered.

Working independently a typical student is able to complete both the Foundation Course and the ECA extension in about one semester if s/he works one hour per day five days per week, on average.

This program is not achievable easily in a standard classroom for reasons fully discussed in Dr. Del's book *Teaching Math*.

"What will this cost?" is the typical next question.

"What would it be worth for a student to be able to pass the ECA Exam and be fully prepared for future math studies?" is a question you should answer first.

It will be less than $100 per student.

For a quote for your group you may contact:

Debbie Goodman at: 812-355-3030 or email:

dgoodman@TriadMathInc.com

General Pricing information for individual students will be available at the Website:

www.TriadMathInc.com/products

Special Message To:

All ADULTS

Do you know that . . .

"For less than $100, and about 50 hours of your time You can become matherate."

"So What?"

"Here's What!"

In the old 20th Century *Second Wave* (Alvin Toffler's paradigm) Economy you only had to be literate to get most good jobs.

In the 21st Century *Third Wave* Economy you must be matherate to get many good high paying jobs. (Definition of "matherate" next page)

You must be matherate to qualify for training in most modern technical fields.

A highly trained technical worker can make a very good living in today's Third Wave economy. $25 to $75 per hour will be common.

An untrained or semi-skilled worker, who could get a good job in the old Second Wave economy, will have great difficulty making a good living in today's economy. Unskilled jobs will pay $10/Hr or less.

Which are you?

Do you want to get a really good high paying job with a bright future?

If so, read the above Headline again.

Then, go immediately to www.TriadMathInc.com to find out how.

The Promise made in this Headline was not possible until the creation of the Dr. Del Practical Math Foundation Course, an Online, Self-paced, Interactive course of just the Right Topics to solve most practical everyday technical math problems.

Definitions in The Free Dictionary by Farlex

I. lit·er·ate
adj.
1. a. Able to read and write.
b. Knowledgeable or educated in a particular field or fields.
2. Familiar with literature; literary.
n.
1. One who can read and write.
2. A well-informed, educated person.

II. math·er·ate
adj.
1. a. Able to solve math problems.
b. Knowledgeable or educated in mathematics.
2. Familiar with mathematics.

n.
1. One who knows and can use mathematics
2. A well-informed, educated person.

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