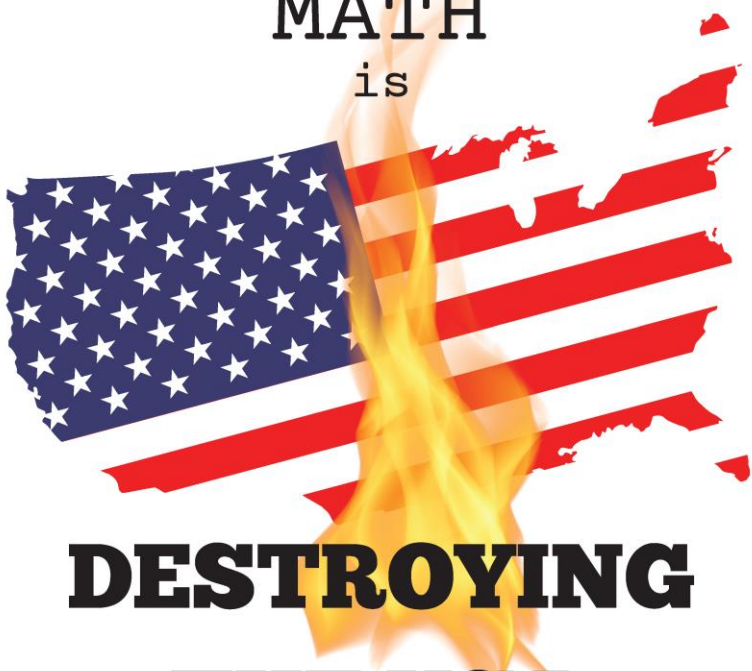
How & Why…

PUBLIC SCHOOL MATH is DESTROYING THE USA



By Craig Hane, Ph.D. aka Dr. Del

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is

Destroying the USA

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Foreword

Four Leaders who inspired me to write this "controversial" book.

"Mathematics is the science that defines the world and the universe using techniques that are commonly called MATH.

There are four branches of MATH
Arithmetic, the study of numbers,
Algebra, the study of formulas,
Geometry, the study of shapes and sizes,
Calculus, the study of quantities and changes.

Some MATH techniques are required in most professions and simply understanding the daily deluge of numbers & percentages.

I want you to learn about Math Education and think how it will affect your life and that of your children.

Think about your own experiences and life, and imagine how it could be today if you had had a different experience.

This book demystifies MATH and shows you how to use this most consequential science as told by Dr. Del's journey."

Rod Harmsworth, Ph.D., Biology and Business

“If You see Fraud and do not say Fraud, then You are a Fraud”

Nassim Taleb. Ph.D., Author of *Antifragile* and other books.

“A man is morally free when . . . he judges the world, and judges other men, with uncompromising sincerity”

George Santayana, Ph.D., Philosopher

"Nobody made a greater mistake than he who did nothing because he could only do a little."

Edmund Burke, Writer, politician, journalist, philosopher

In a Nutshell

Public School Math . . . is . . . Destroying the USA?

“Hyperbole?,” as your favorite narcissist might say.

Not really!

“Destroying how wonderful the USA could and should be.”

How?

The Economy.

Many wonderful high tech jobs are going unfilled because our Public School Educational System is NOT teaching the Appropriate Math successfully to our young students and future workers.

This is resulting in lost productivity and, thus, loss of better higher quality products and services the USA could have.

This results in workers making a lot less money than they would IF they could fill these high paying jobs.

This results in poorer quality of life for the workers and their families.

This results in greater inequality of incomes and financial resources.

This results in much worse Tribal fights in the USA.

This results in a much poorer Society in the USA.

Argument Synopsis

Public School Math ——> Weaker Workers ——> Worse Families ——> Weaker Businesses ——> Worse Economy ——> More Tribal Conflicts ——> Worse Society——>

Destroying How Wonderful the USA Could Be!

This Book provides the detailed arguments and then a Solution.

An Optimal 21st Century Math Education

Available to All Students Today!

To get started, go to Special Offers on p. 163.

Overview and Some Advice

What is the best way for You to read this book?

Clearly, it depends very much on You and your background and interests.

Here is some advice I hope might help you.

If you're in a hurry, just go to Part IV, Chapters 19 – 22 to see the Problems with Public School Math, and then Part V to see the Solution.

But to get a full picture of how Parts IV and V came to exist, you might enjoy Parts I, II, III.

Here's what's in the Six Parts.

Part I is intended to just get you to think about:

1. What Math Education Meant to You.
2. What You Think Math Education Should Mean to a Student today.
3. Math Education's effects on our Economy and Society.

Part II is my story to show you how my early life from Pearl Harbor through High School began to evolve the Solution I explain in Part V.

Part III is how my story through College and up to entering Graduate School continued the evolution of the Solution.

Part IV Chapters 19 – 22 explain the situation with Public School Math today.

Chapters 23 and 24 are more about my continued evolution to the Solution.

In Part V, Nine Chapters explain the Solution in depth and also give you a lot of references to various Resources to help You deliver an Optimal 21st Century Math Program to any Student.

Part VI is a Summary and Conclusions.

So Chapters 19 – 22 and 25 -33 are the Meat of this Book.

If you agree with me, then I invite You to join me in pursuing my Mission to Deliver an Optimal 21st Century Math Education to all Students.

Go to Special Offers on p. 163.

Part I:

Current Public School Math

“Facts of Life”

Chapter 1: Your Math Education Experience?

Of course, Your Math Education Experience is unique.

Was it all good? All bad? Some of each?

The Good, the Bad and the Ugly!

Were you ever in a Math Class where you couldn't understand the teacher? If so, how frustrating was that, and what did you do about it?

Did you ever feel 'stupid' or 'inadequate' when some other student seemed to always know the answers the teacher asked and you didn't?

Did you ever feel you were the smartest kid in the class?

Did you ever have a one-on-one Math Tutor?

If so, how was that experience?

Do you like math? Love math? Hate Math? Afraid of Math?

Do you believe Math can empower a person to learn other things and do some things otherwise unattainable?

Could You have a more fulfilling or better life IF you were better at Math?

These questions were just to stimulate you to begin to think about Math Education and how it has affected you.

But more importantly, how Math Ed is affecting today's students.

How would the students you know answer these questions?

My hope is that a student would answer all of these questions in a very positive and uplifting way, and be very happy and satisfied with their Math Education.

In this book, I want you to learn about Math Education and think about how it could, and should, affect your life or other people's lives, especially your loved ones and children.

Most importantly, I will be discussing why today's Public School Math Programs are not yielding satisfying answers to the above questions for many students.

Finally, I will be telling you about a Solution for this Problem and how, today, in this 21st Century, it is possible to deliver a great Math Education to all students.

I will be telling you my story too, along with my answers to all of the above questions.

Some of them might surprise you.

Finally, I will tell you about a Solution to this Challenge of how to deliver a Successful Math Education Program to all students.

Plus, How this Solution evolved over time.

First, let's be sure we agree on the current situation with Public School Math.

That's what the next five Chapters in Part I discuss.

If you just want to get to the “nitty-gritty,” follow the Advice in the Overview.

Chapter 2: How Most Students Feel in Math Class

Most students don't feel good in a Math class.

They don't really understand everything the teacher is 'explaining', especially in a lecture format.

Once You get lost, it's hard to 'catch up'.

If the teacher asks a question, usually some student quickly answers it, and You feel 'stupid'.

Why didn't I know that?

Am I simply 'dumb', mathematically speaking?

Is your Math homework fun? Easy?

Something you look forward to? Enjoy?

For most students, the answers might have been Positive at some point in their educational journey, but then in some Math Class, the answers became Negative!

Do you see the value in the Math you are being asked to learn?

Many students don't and for good reason!

It won't be of value or help to them in their future.

Will you ever need to know that the square root of 2 is an irrational number?

Will you ever need to solve a Quadratic Equation?

When, where or why?

The answer to these two questions and many similar ones… No for most students!

The Bottom line: Most students have had a bad experience with Math, and even if they survived it, they find much of what they were tested on of little value.

Indeed, how much of the Math you have been tested on do you remember and use?

Years later, is there some Math you wish you had learned that would be of value to you today?

What are the Results of today's Public School's Math Programs?

The answer is in Chapter 3.

Chapter 3: Fear and Frustration - Lack of Self Confidence and Esteem

In my experience both as a math student and math teacher, I have found a large majority of students have had a less than good experience in some Math Class.

Fear and frustration are common when a student is struggling to understand the concepts and do the homework problems, all while observing that some other students seem to be doing well.

Either you understand a Math Concept and can solve a Math Problem or not!

It's an A or F. Pass or Fail for One Math Concept, Math Tool or Process.

There is really no such thing as a C in a Math Subject except for the phony ways it is usually graded in our Public Schools.

This will be explained in Chapter 22. It is clever and diabolical.

My Belief: If a student has Fear, Frustration, Lack of Confidence and Lack of Self-Esteem in a Math Class, then the Math Class has failed the student. Do you agree?

Good teachers know this, but often can't really do anything about it in a Classroom setting as will be discussed in Chapter 19.

The Good News is that this is all so Unnecessary in the 21st Century.

NO Student should have Fear, Frustration, Lack Self-Confidence or Lack of Self-Esteem due to Math "failure."

Part IV will explain How and Why this often happens in our current Public School Math Programs.

Part V will reveal a wonderful Optimal 21st Century Solution.

Chapter 4: Lost Opportunities

There can be a great career opportunity in an Apprenticeship Program.

There can be a great career opportunity in a Technical Field.

There can be a great career opportunity in the Military.

To take advantage of such a great career opportunity you must be Literate, i.e. Speak, and Read, and perhaps, Write a Natural Language like English in the USA. This is nothing new and was expected and pretty well handled by our Public Schools. This was true for the whole 20th Century.

However today, in the 21st Century, to take advantage of many such great career opportunities you must also be Matherate, i.e. know the necessary Practical Math, which is a Universal Language everywhere including the USA.

As the USA moves rapidly into the Post-Industrial Economy with Robots, A.I., Computers, and a plethora of new technologies, appropriate Math is necessary to learn the many other things required in the 21st Century Technical Workforce.

“Plethora” means - “Many” - “Lot's of”- “Millions” etc.

Example: Peter Zeihan says Jobs like Plumbing, in our modern economy, will be worth a high yearly income, like $85,000 in a middle cost of living area. Ditto Zeihan says, many other Jobs like Micro-Manufacturing and Custom Manufacturing, which now require mastering Modern Tools and Technologies.

These Millions of Technical Jobs going unfilled in 2022 require the Worker to learn and master some Technical Knowledge and Skills, which are impossible to acquire without Proper and Adequate Practical Math Knowledge.

I will discuss the required Practical Math Foundation in depth in future Chapters like Chapters 21, 28, 29, as well as How any Student can become Matherate today, very cost effectively, in terms of both the Student's Time and Money.

This is where our Public Schools are Failing, Big Time!

How and Why are they failing?

That is answered in Part IV.

What Effects are these Lost Opportunities having on the USA?

That's what we are exploring in Part I.

Then in Part V, you learn about a wonderful 21st Century Solution.

Chapter 5: Weak Economy

What makes a Strong Economy?

The Ability of Businesses to obtain capable workers who can create and deliver and produce great products and services.

Today, (2022), it is reported that there are over One Million Jobs going unfilled due to a deficiency of qualified capable workers.

This results in a Weaker Economy compared to what the Economy could produce IF these Jobs could be fulfilled.

These Jobs pay high wages far above the mandated "minimum wage."

There are many workers struggling to survive on near minimum wage jobs. So, why don't these low paid workers take these wonderful high paid Jobs?

A Lack of proper Education and, thus, Abilities.

For Technical Jobs, this means the Practical Math Foundation necessary to then learn the various technical things necessary for the Technical Job.

Practical Math is like a Foundation for a Building.

If you build a Building on a bad Foundation, what is the result? Over time the Building will not do well.

Practical Math is a Good Foundation both for the learning of More Advanced Math and/or for pursuing a Technical Career.

You can't train a new worker in technical subjects IF the worker does not have the Proper Math Foundation, or Practical Math.

Our Public Schools are not teaching our students the Proper Practical Math Foundation. Thus, the Public School's Math is causing the USA to have a much Weaker Economy than it could and should have IF our students were being taught a Proper Math Foundation, or Practical Math.

Part IV will explain this in detail and Part V will show You a 21st Century Solution to this horrible Dilemma.

Indeed, in today's 21st Century, any Student can Learn Practical Math in less than One Year thanks to many wonderful technologies.

That will be explained in depth in Part V.

Jump ahead to Chapters 28 and 29 If you just can't wait!

Chapter 6: Weak Society

What makes a Strong Society?

Members who are happy and get along together! If a worker is over stressed and feels underpaid, is s/he happy? If a worker can't provide for himself or herself, or worse yet, provide for his or her loved ones, is s/he happy?

Which is a Stronger Society?

Tribe A: 80% Happy Workers and 20% Unhappy Workers

Tribe B: 20% Happy Workers and 80% Unhappy Workers

Tribe A or Tribe B?

If you answered Tribe B, I suggest you quit reading this Book.

Obviously, Tribe A is the answer I believe in.

What if 80% of your potential Technical Workers cannot get a Job because they don't have an adequate Practical Math Foundation to enter the Technical Workforce, and they are forced to take some low paid, unskilled labor job?

Hmm. Would you have a lot of Unhappy Workers? Tribe A or Tribe B?

Today in this 21st Century, our Public Schools are Failing to provide their students with an Adequate Practical Math Foundation for them to then successfully enter the Technical Job Workforce.

It is just a Sad Fact resulting in a much Weaker Society than the USA could and should have.

That is what I mean by, "Destroying the USA!"

Fortunately, there is a 21st Century Solution to this Problem! However, this Solution will destroy the current Math Education System which permeates our Public School System.

It will probably be a Big Fight, or War, to change our Math Educational System. Either Public Schools will have to change their Math Education Programs or they will just fade away.

Homeschool, Private Schools and Charter Schools might be the future.

Hopefully, Public Schools will transform and do the Job they need to do.

In Parts II and III, I will tell you my story, if you are interested in How and Why this all came about.

I, too, was a Victim of the horrible Public School Math Programs.

However, I was "saved" by three wonderful teachers. I was very fortunate.

My decades of experience in Math Education and Math Applications has led me to be able to take advantage of some wonderful modern 21st Century Technologies to create an Optimal 21st Century Math Program you will learn all about in Part V.

To see how this all came about, continue into Parts II and III.

To get to the "nitty-gritty," jump ahead to Part IV.

Part II:
Dr. Hane Stories
(Solution Began
with Pearl Harbor)

Chapter 7: How and Why I Loved Math Through the 8th Grade

I was born in November of 1938 in Greencastle, Indiana.

My Dad (father) was Delbert G. Hane and Mom (mother) was Ruth Mary Haines Hane. My Grandparents were Claude and Lena Hane and Ray and Golda Haines. I also had many Aunts and Uncles.

One week after my third birthday party, something happened they all called “Pearl Harbor.” I didn't know what it was, but could sense it might be a Big Deal.

Indeed, it was! Everyone's life was impacted in various ways.

My Aunt Inez Hane Davis (age 23) married my Uncle Jack Davis (age 53) in 1933 and they built a Log Cabin on a 10 acre farm on the banks of Deer Creek, 5 miles south of Greencastle on State Road 43.

Uncle Jack was a popular Barber in Greencastle, as well as a Builder and a Farmer. Aunt Inez was a homemaker. They never had any children and Uncle Jack lived to age 80.

For some reason, Mom and Dad decided to build a small wooden frame house on 2 acres of Uncle Jack's farm between Uncle Jack and Aunt Inez's Log Cabin and the banks of Deer Creek in about 1941. Maybe because of Pearl Harbor?

Dad took a job in a War Plant, International Harvester, in Indianapolis, Indiana about 1.5 - 2 hours from our house. So he was gone 12 or so hours per day, and slept about 8 hours when he was home, so I didn't see him too much.

Mom was a beautician and worked in a Beauty Shop in Greencastle, so I didn't see too much of her either. They raised some chickens and a garden to eat in the summer. I played a lot by myself in the Woods and Creek.

Aunt Inez became a second mother to me and grandma Golda Haines helped raise me too. I spent a lot of time with them and learned a lot of practical things from them.

Uncle Jack became like a second father, and I was also close to Grandpa Ray Haines, who worked on the Pennsylvania Railroad. He repaired "hot boxes" which were the bearings on the Railroad Cars.

We had no running water, central heat or a telephone.

But we did have a two-holer outhouse, cooked on a kerosene stove and heated our house with a pot-belly coal stove, though we did have electricity and a radio.

I roamed the woods playing in Deer Creek and spent a lot of time with Aunt Inez and Grandma Golda.

Life was wonderful for me!

So, what does this have to do with Math?

Well, during WWII, there were no Preschools of any kind. School started in the First Grade at age 6. Few students received any prior formal education.

In 1942, "Cheerioats" cereal came out and Uncle Jack started using Cheerios to teach me to count and add small numbers. When we got to 10 and above, he started using pennies and dimes too.

By the time I was five years old, I had learned to count with the decimal number system into the hundreds, all thanks to Uncle Jack. Homeschool!

Of course, he taught me a lot of other practical "farm stuff" too. I explain the Math he taught me in my Uncle Jack videos for parents even today.

In addition to the decimal number system, Uncle Jack taught me practical geometry in later years. For example, how to use string to make sure a fence post was Square to the ground with the 3,4,5 Rule. Or how to make a third of a Square Angle with the 1,2 Rule.

Uncle Jack was not a math teacher. I doubt he ever heard the words Pythagorean Theorem. But he knew the 3,4,5 Rule for a Square Angle.

He was a Barber and Builder. He had built his Log Cabin, Barn and Outhouse, and it took a lot of Practical Math to do this.

My Dad decided to enroll me in the first grade in the four room school with 8 grades in Putnamville, about three miles to the west on US 40. There was a school bus to get there. I was five years old and the youngest kid in my class.

Miss Bernice Lewis was the teacher of the 1st and 2nd Grades all in one room.

I was very bashful and couldn't skip, sing or color well since I am color blind. So, I was clearly the "dumbest" kid in the class. Everyone knew it.

Needless to say, I was not very happy at school at first. But, so what? Most adults weren't happy either. WWII had started. Rationing. Draft. War Plants. Life was challenging for everyone!

One day Miss Lewis put all of us first graders up front and decided to teach us to count. Due to the war, very few kids had been taught anything about math. Most first grade students struggled to count to ten.

I know, hard to believe today, but this was 1943 and the War was not going well for the USA!

When Miss Lewis got to me, thanks to Uncle Jack and homeschooling, I started counting and when I got into the 60's, Miss Lewis stopped me with a surprised look and asked how far I could count.

“I don't know,” I said. “Just on and on, I guess.”

Well, Miss Lewis sent me back to my desk to practice coloring. Hmm. Maybe I wasn't so dumb after all.

The next day she asked me to help her teach my classmates, who were older, larger, and smarter in all other subjects than me, to count. I was happy to do so and help them like Uncle Jack had helped me.

For the first time, I was admired by my classmates. Some new quality or feeling set in for me. Self-Esteem?

I loved teaching them sort of like Uncle Jack had taught me using stones, marbles and other things like Cheerios.

I would also follow along the Second Graders as Miss Lewis taught them Math. By age six, I had learned all the Math through the Second Grade.

The next year, I helped Miss Lewis teach my classmates Math, some of which she was learning too, thanks to what Uncle Jack was teaching me at home.

Uncle Jack enjoyed my stories about what was happening at school. He taught me more and more practical Math that he used in Building. He tutored me using what today I call SPIKE Pedagogy (see Chapter 26).

It was very Practical Math Content that everyone knew was valuable for all sorts of things. Numbers, Geometry and their applications to building.

I had very poor math teachers in 3rd and 4th Grade. For example, Mrs. Groner, my 4th Grade teacher, taught us that one million was two thousands... the first three 0's are one thousand and the second three 0's make two thousand which makes 1,000,000.

Riding home on the school bus, I thought my Family were millionaires since I was pretty sure we had saved two thousand dollars. My Dad quickly set me straight on this. Uncle Jack and my Grandfather Ray thought this was pretty funny!

Uncle Jack's Practical Math served me well all the way through the 8th Grade.

We had moved into Greencastle when I entered the 7th Grade in 1950. WWII was over and the USA had won.

Dad started selling Ford cars and trucks while Mom opened up Ruth's Beauty Shop. We lived in an apartment above a restaurant next to DePauw University in Greencastle, right across the street from the Sigma Nu Fraternity House.
Now we had an indoor heated bathroom. Wonderful!

But I had left my idyllic life in the woods and Deer Creek and Putnamville. I was not happy. I missed my wonderful life of childhood, the woods and the creek, while I was entering early adulthood.

I took various jobs and started saving money. I set pins in a Bowling Alley and delivered Newspapers in the morning.

In Greencastle we had different teachers for different subjects. I had Mr. Irwin for 7th and 8th Grade Math for PreAlgebra and I did fine, thanks to my Uncle Jack's Practical Math.

Then the 'Stuff hit the Fan' in my Freshman year in High School, or 9th Grade.

It was the Worst year of my entire Education Career!

Wonder why?

Math!

Algebra 1.

Chapter 8: How I 'Flunked' Algebra 1 in the 9th Grade

In Putnamville, I once heard our 8th Grade teacher ask my 6th Grade teacher, "What is this X in Algebra?" Neither knew. A mystery!

Upper level math, Algebra, Geometry and Trigonometry were a Great Mystery. Something to fear.

So, in the 9th Grade at Greencastle High School (GHS), I was enrolled in Algebra 1.

My teacher was Mr. Harold Hardman, the GHS Math Dept. Chairman and Algebra and Trigonometry teacher. A Math Expert! So, everyone said.

Mr. H would tell us something like a formula or process and tell us to memorize it. Then do a bunch of problems. If you made a careless error. Wrong!

If you asked how or why something like a formula worked, he would tell you to just learn it and use it and to be quiet.

I really wanted to know where the Quadratic Formula, so mysterious, came from.

"Shut up and learn it and apply it! That's Math.," quoth Mr. H.

I make a lot of careless mistakes when I work fast under pressure. Always have and always will. I can catch them and correct them when time allows. But, on a timed test I didn't have time to catch and correct them all.
So, guess what?

I scored poorly on most of the timed Algebra tests.

The Result?

A bad grade for me, like a C or B.

I “Flunked” Algebra!

Bottom line! I did not like Algebra. And that's an Understatement.

Uncle Jack couldn't help me. He didn't know anything about it.

And I always made careless errors on a timed exam.

For the first time in my life, I did not get an A in Math and was not near the top of the class. Some other students made less careless errors than me and got an A.

So I did not do well in Algebra, and for the first time in my life, I did not like Math. I was depressed.

No one in my family had ever gone to college. I was told from an early age by my family that someday I would go to college, and this was the “Open Sesame” to great success in life.

I believed it! I would go to college.

My GHS Counselor, Mr. Skelton, explained to me that since I had not done well in Algebra, I would not be able to go to College. “Take Shop. You're going to be a working man like everyone else in your family. You obviously are not college material,” he said.

Shop was Industrial Arts. I took Shop with Mr. Stewart. I did really well thanks to my Uncle Jack's Practical Math training.

But No College in MY Future!

UGH!

The summer between my Freshman and Sophomore year was pretty depressing.

I had hit a Math Barrier. Algebra! Game over!

At least I had a lot of company. Most students had not done any better in Algebra than I did. None of my friends were depressed like me. They had always struggled with Math. They didn't have an Uncle Jack in their lives.

It's tough to go from being one of the best and helping teach others to being mediocre. Oh well. Life goes on. Just mow some lawns, deliver some newspapers and save some money.

Forget Math and forget College!

Then a "Miracle" happened, or so it seemed to me!

Chapter 9: Why I Loved Geometry and Did Well in the 10th Grade

Sophomore Year. 10th Grade. Geometry.

Miss O'Hair was an "old maid" school teacher who taught Geometry at GHS.

Mr. Hardman was the Dept. Chair and taught all the other Math, Algebra 1 and 2, Solid Geometry and Trigonometry.

Euclidean Geometry can consist of proving a lot of Theorems about various geometric objects like lines, triangles, circles and combinations of them.

It is very logical. You can create Proofs of some Statement. You prove something by going Step-by-Step from certain assumptions to arrive at a final statement. A Basic "common sense" Logical argument.

It starts with certain Assumptions called Axioms and Postulates and then you prove more statements called Theorems. Then one Theorem leads to another Theorem, on and on forever.

When taught this properly, Geometry is pretty easy to learn.

By the way, this is NOT how I teach Geometry today, but that's another story.

Miss O'Hair was, for me, a very good teacher, like Uncle Jack.

You try to understand each concept and the rules of reasoning.

Long story short, I really liked Geometry and loved proving theorems.

I was not a particularly good student in other subjects, usually getting by with a B or so. This was especially true in subjects I didn't find interesting.

I worked especially hard in Geometry and was at the top of my Class again. Miss O'Hair would have each student go to the front of the class and explain how to prove some statement, which was a Problem in our book.

Your grade depended on how many demonstrations you delivered to the class. You got double credit if you did your demonstration without notes, which I always did. This required really understanding your Argument or Proof.

The student with the fewest points would be given the opportunity to do a problem demo. Since I soon had the most points, I would have to wait until we neared the end and reached the hardest problems. So I would work those problems first and then be the only student who had gotten that far.

Result: I got an A+ in Geometry and went to a Geometry Contest.

Mr. Skelton was surprised. I was elated and had recovered some of my Self-Esteem and Confidence.

So now back to Mr. Hardman for Algebra 2 in 11th Grade.

What do you think happened?

Mr. H again. Fun and Games. Life can get interesting.

I'll bet you can predict what Happened my Junior Year.

I became a Troublemaker, or so Mr. H thought.

Chapter 10: BIG Problem – Algebra 2 in the 11th Grade

I was now somewhat of a Hero in Math thanks to Miss O'Hair and Geometry.

Then I became a "trouble maker." When Mr. Hardman taught us some algebraic processes, I asked him to Prove it worked. He did not like that and usually refused.

To give him some credit today, I realize he probably simply did not understand it. He believed that for Math, you simply memorized some Formulas and Algorithms, applied them and didn't worry about where they came from.

I still could not understand where the Quadratic Formula came from. I doubt if Mr. H understood the "Complete the Square Process" it came from.

And I still made careless errors on timed tests.

The Result: Algebra 2. Grade B. Still not College Material according to Mr. Skelton.

I wonder how many GHS students were in this category over the many years at GHS? I know a lot of my fellow students were. And, they were just as "smart" as me. But they did not have an Uncle Jack in their lives.

Many of them didn't even take Geometry with Miss O'Hair. And the other Geometry teacher was horrible, as my younger brother found out a few years later. He is just as smart as me and he never even considered going to College!

So Mr. H, and GHS, did have another Geometry teacher who was as poor a teacher as he was! I'm not sure how Miss O'Hair survived at GHS.

GHS was considered a good high school. It was probably average and typical.

Then, a second Miracle for me!

I doubt you could predict this.

I certainly didn't! See Chapter 12.

But first a second amazing thing happened to me. See Chapter 11.

Chapter 11: The Fearless Freddie U Story

One of my best friends at GHS was the Captain of our Football team.

He was “Fearless Freddie U,” aka Fred Unsworth, who lived in the upper class part of Greencastle in Northwood.

His Dad was a successful business man and drove Fred to become a good football player since he was too short to play basketball. Fred worked very hard at physical fitness and football.

Fred struggled with Algebra 1 like most GHS students, but got through it by 10th Grade with a Grade of C-. Then, the Geometry class he was taking in the 11th Grade with Miss O'Hair was very difficult for him.

In all due respect to Miss O'Hair, when you have a class of 25 or 30 students, you just teach to the top few and simply don't have time to help the slower students like Fearless Freddie and many others.

Freddie was flunking Geometry with Miss O'Hair. Betty Unsworth, Fred's Mom, knew I had done well in Geometry and asked me if I would help Fred. Of course I would be happy to. He was one of my best friends.

Then Betty said she would pay me 75 cents per hour to do so on a regular schedule. That's about $8 an hour in 2022.

I told her I would do it for nothing since Fred was a good friend and I always helped classmates when I could. But Betty knew I worked very hard at many part time jobs and was saving money, and they had plenty of money.

So I tutored Fred, one-on-one using what today I call SPIKE Pedagogy (See Chapter 26).

He went from a Grade F in Geometry to a Grade B. Everyone, including Fred, was amazed. How could it be?

No Classroom teacher can apply SPIKE Pedagogy to the slower or less prepared students in their class. A good teacher knows this, but can't do anything about it, as I learned five years later to my great chagrin.

I'll tell you all about it in Chapter 17.

But first, “My 2nd Miracle Year.”

Chapter 12: THE Miracle – College Algebra in the 12th Grade

Mr. Hardman taught Solid Geometry and Trigonometry for the Seniors at GHS.

I now knew that he was not a good math teacher, for me at least. So I went to Miss O'Hair for advice.

I still wanted to go to College but was told my Counselor, Mr. Skelton, and Principal, Mr. McCammon, that I simply was not College material, thanks to Algebra. Plus, I made a lot of C's and B's in other subjects as well.

DePauw University is a small liberal arts college in Greencastle and was highly regarded in Indiana as a very good college. Indeed, DePauw and Rose Polytechnic Institute of Technology in Terre Haute, were considered the best private colleges in Indiana. Well, Notre Dame was the best, I guess.

Miss O'Hair suggested to me that perhaps I should take an Algebra course at DePauw, as a Special Student, to see if I could do any better in Algebra.

WOW. No one had ever done that. Out of the Box, big time!

Mr. McCammon, the Principal, was adamantly against it. He told me I would embarrass GHS. After all, I had done poorly in Algebra at GHS. Thus, I would do even worse in a good college like DePauw. They had a Formula!

My Dad, 8th Grade education and all, had a talk with Mr. McCammon and to my surprise Mr. McCammon gave me permission to embarrass myself and GHS.

So, I enrolled as a special student where the DePauw registrar made a mistake. I should have been enrolled in a DePauw freshman Algebra course, Intermediate Algebra, but they enrolled me in a sophomore Algebra course, College Algebra.

My professor was a tall droll man, Dr. Clint Gass, chairman of DePauw's Math Dept. He was unlike any teacher I had ever had. Dr. Gass pointed out to us the first day the similar spelling to the greatest mathematician of all time, C.F. Gauss. I didn't get it. I was 16 years old and an unsophisticated kid in a class of large mature, sophisticated DePauw Sophomores.

There was a wonderful female student who treated me like a kid brother and sat next to me. She was smart as a whip and we helped teach each other Algebra along with Dr. Gass.

The first test came... a timed exam. Ten questions and show your work.

Dr. Gass was a great teacher like Miss O'Hair and I really understood the Algebra concepts and Algorithms (Processes to solve Algebra Problems).

But, I still made careless errors on a timed exam. On the way home after the Test, I reviewed it in my mind and identified at least four careless errors. Wow.

Mr. Hardman would simply mark a ten-point problem wrong if you didn't get the right answer. Careless errors were not permitted. So, I knew I had flunked the first exam.

Mr. McCammon was right. Embarrassing. Depressing!

Dr. Gass wrote all the class's test scores on the blackboard. 97, 95, 94,.....,80's...70's... 60,60,60,..55, 52, 47, etc. Four 90's were A's. Six 80's were B's, etc. 70's were C's and 60's were D's. The 50's and 40's were F's.

I prayed I would at least get a 60 and pass the exam. I was very depressed. Ugh. Failure! Dr. Gass handed back our test books with the scores inside.

My “big sister" friend that sat next to me was very smart and opened hers. She got a 97. No surprise. “What did you get Craig?” she asked.

I was simply afraid to look and certainly didn't want her to see my failure. Finally, she prodded me to open and look.

Unbelievable! 94 Grade = A. What? How?

Dr. Gass would deduct 1 or 2 points for a careless error in a ten-point problem if your work showed you really understood it. In fact, he missed one of my careless errors.

Suddenly I was elated. I have never in my life gone from such a depression to such an elation in a quantum leap. My life changed, for the better, that moment in time.

After class, I went to Dr. Gass and pointed out the careless error he had missed. He thanked me and smiled and said, “Don't worry about it.”

Dr. Gass became my Mentor and was a Huge Influence on my life as you will learn in future Chapters.

I ended up with an A in his College Algebra class at DePauw. When I told Mr. McCammon and Mr. Skelton this, they were incredulous. How could it be?

I told them that Mr. Hardman was simply not a good teacher.

Oh well, he was a tenured chairman of the Math Dept. Nothing changed. How many GHS students' lives did he ruin like he almost ruined mine?

Without Uncle Jack, Miss O'Hair and Dr. Gass I would have become a working man just like my family members and my brother.

And now the rest of the story begins!

Chapter 13: My First Classroom Teaching Experience

In the Second semester my Senior Year at GHS, I decided to take Trigonometry at DePauw even though it was being taught at GHS by Mr. Hardman. If you have read the previous Chapters, the reason is obvious.

I had Mr. Johnson as a professor and he was pretty good too. Also, DePauw had a three-week head start ahead of GHS.

One very important part of Trig is proving what are called Trig Identities. This is sort of like proving Theorems in Geometry. So, I had learned this at DePauw with Mr. Johnson and was pretty good at proving Trig Identities. They are Easy for anyone to learn IF taught properly.

At GHS, Mr. Hardman had just started teaching Trig Identities to his students, which included both Juniors and Seniors since he only taught Trig every two years.

Mr. H got the flu and was going to miss a whole week of classes. Mr. McCammon asked Miss O'Hair if she would teach Trig for the week. She said she would, but only IF Craig could teach the Trig Identities Proofs since she had not taught them for many years.

McCammon agreed and so I entered the GHS Trig class with all of my classmates to teach them how to prove Trig Identities.

At first, I was in such a state of fright that my knees were weak. I stood in front of the class and looked at my curious classmates. Craig's teaching us?

I asked my classmates which Problems they had questions on. I had worked them all out in preparation for the class. Junior basketball star, Bergie Remsberg, said “Sure Craig, Problem #1.”

Turns out Mr. H had not really taught them anything about how to prove a Trig Identity. So, I walked Bergie and the class through the Proof by asking them a series of questions until they understood whatever it was they were trying to learn.

I call it the Socratic Technique and always use it when teaching math. This forces and empowers the student to understand whatever you are teaching step-by-step.

Lectures rarely do this. 80% or more students usually get lost sometime during a Lecture. Most won't admit it and many never recover.

By the end of the week, the GHS students had proven ALL of the Trig Identity Exercises. Yes, I helped them with the Socratic Technique. But they learned it by doing it. Some of them loved it as much as I did. Most of them were just as capable and smart as I am.

When Mr. H returned, I was told he asked the students what they had done the week he was gone. They excitedly told him they had learned how to prove Trig Identities and that Craig had taught them and they were ready for the test.

Believe this or not! They told me that Mr. H. told them that Trig Identities were not important and there would be no test on them!

At the end of that year, those students, my classmates, signed my high school yearbook and most said I was the best Math Teacher they had ever had. I have never been so elated, humbled and thrilled.

Unfortunately, none of them had learned very much Trigonometry, other than Trig Identities. And Trig is one of the most important subjects in STEM for Science and Engineering.

I decided I wanted to learn more Math and what I really enjoyed was then teaching Math to other students.

So, College here I come. But, where? What College?

Something pretty amazing happened that was life changing for me.

And, just maybe for You too, or at least some Student you care about!

Chapter 14: Off to the #1 Liberal Arts College in the USA

I had worked at many part-time jobs and projects for six years and saved about $2,500 by the time I graduated from GHS in 1956. That's about $25,000 in 2022 dollars.

At that time, tuition to a good liberal arts college was only about $600/Year. I know, that is only $6,000 in 2022 Dollars yet today's tuition is about $60,000. So there has been a TEN times inflation beyond the normal inflation in higher education and private schools.

I had a B average at GHS and was not even elected to the National Honor Society by the teachers, like virtually all the other college bound seniors. I played no sports since I worked all the time and didn't like athletics anyway.

But the DePauw professors I knew thought I could get into DePauw thanks to Dr. Gass. Maybe on a Rector Scholarship. However, DePauw had a lot of Greek Fraternities and Sororities and I had known a lot of unhappy DePauw students because of this for various reasons.

I did not like the Greek life and decided to not go to DePauw. Also, for various reasons I wanted to escape Greencastle. I had not enjoyed my six years there for many reasons, not necessarily Greencastle's fault.

So I went to my now Mentor, Dr. Gass, and asked him to recommend some small coeducational liberal arts colleges I might go to. He gave me a list of five no one in my family, including me, had ever heard of.

Swarthmore, Oberlin, Carleton, Reed and Pomona.

I did not realize that these were the top five USA Liberal Arts Colleges in 1956. Oberlin was #1 and Swarthmore #2. I went to the library and looked them all up. Oberlin was the closest and I could get there via the Penn Railroad direct from Greencastle.

So I visited Oberlin in the spring of 1956 during the GHS Prom weekend. I loved Oberlin and the students I met. So I applied to Oberlin.

I was working in Gardner's Food Market and helping on the Checkout Counter when the Chairman of the DePauw Philosophy Dept., Dr. Compton, who knew me, asked me where I was going to college. I think he expected me to say DePauw.

I said, “Oberlin College.” Dr. Compton looked at me very surprised and asked, “Have you been accepted to Oberlin?”

I replied, “No, I just applied two weeks ago.”

He replied back with a smile, “Well, Craig, don't get your hopes up too high.”

It had never occurred to me that I might not get accepted. What did he mean?

Well, Oberlin was the #1 Liberal Arts College in the USA at that time. Thanks to a very large endowment by Charles R. Hall, the Alcoa guy who discovered how to recover Aluminum from Bauxite ore while at Oberlin. It is spelled Aluminum in the USA because of a printing error on the first batch.

Anyway, it turns out that Oberlin had quotas for enrolling students. For example, a Jewish female from New York had a lot of competition and would have a very hard time getting in. Jewish Males were similar. I found this out from my many wonderful Jewish Oberlin friends.

However, a Christian Male from Indiana had little competition. So, I was admitted to Oberlin in spite of a low GPA, no Sports and no rich relatives. I have always believed that Dr. Gass wrote them a powerful recommendation which was why I was admitted.

So, September 1956 and off to Oberlin College at 17 years old.

Looking for Adventure.

No Greek. Very liberal. Great professors. Great guests like Martin Luther King, Ayn Rand and Hal Holbrook, to name a few I met and talked to.

I even participated in sports for the first time since I was now nearing physical maturity and Oberlin was not a very athletic school.

Thank you Uncle Jack, Miss O'Hair and Dr. Gass.

Without these three great math teachers, this would NOT have happened.

I was very lucky and blessed.

And this was just the beginning!

Part III:

The Evolution of a Solution

Chapter 15: Tutor and SPIKE Pedagogy

SPIKE is an acronym I created for a Successful Math Pedagogy:

Self-Paced – **P**roper Content – **I**nteractivity –
Keeping Score – **E**mpathy

You may learn all about it in detail in Chapter 1 in either of my two books:

How and Why Homeschool Math can be vastly superior to Public School Math - OR -

Golden Rule Math for All God's Children

You may get either of these books for Free by going to:

www.CraigHane.com/book

How did I “discover” or “invent” this SPIKE Pedagogy?

It started with Fearless Freddie U I told about in Chapter 11.

Why was I successful teaching Freddie Geometry and Miss O'Hair was not?

After all, Miss O'Hair was my Great Teacher as explained in Chapter 9.

Well, I would simply lead Freddie through a proof Step-by-Step at his own Pace. I would be sure he understood a Step before we went on. Sometimes, it took several discussions and Q & A's before he understood a step.

When he made a mistake or "stupid answer," we would laugh together about it.

I call that Empathy. We all make a lot of mistakes. We learn from them.

Do Not feel bad when you make a mistake. Learn from it. Laugh about it.

When he was not ready to understand a Step in a Proof, we would review previous material he had forgotten or not understood before, until he understood it, and Then move on.

I call that Proper Content. You must understand necessary Prerequisite Math Concepts before you can understand a current one.

We often forget something we once learned and knew. That is why I always encourage my students to do a lot of Review.

I often have to "Learn" something several times before I remember it.

It is also very desirable to show the student why the Math s/he is learning will be of value and interest to them in the future.

Sometimes, I could find an example from football or carpentry Freddie was interested in where the Geometry would be of value.

Uncle Jack always did that for me when he taught me Practical Math.

Finally, after a Tutor session, we would review what new things Freddie had learned.

Keeping Score. What Mistakes he had made and then learned from. Praise for Success in Climbing the Ladder. Praise for Effort. A form of Coaching.

I was teaching Freddie things I had learned only a year earlier myself. I, too, had made mistakes and had to struggle to learn certain concepts. I could tell Freddie stories about how I was struggling with Math myself. Remember, I was a Junior taking Algebra 2 and struggling myself.

So think about it: S P I K E

Each of these was a necessary ingredient in having success Teaching Freddie Geometry.

Miss O'Hair could not take the time with Freddie, or most of her students, to do all of these things. AND it was NOT Miss O'Hair's fault. She was a 'victim' too!

No Classroom teacher in a Public School can teach with SPIKE Pedagogy to all the students in a large class, if any. Since all students have different backgrounds and abilities, it is never possible to really do all of these things with more than One Student at a time.

Furthermore, some Students like to compete and “kid” each other. Hey, Sally got the problem done, and David didn't. David’s dumb. Sally's smart. Very BAD Empathy. Creates Fear and Dislike of a subject.

Now on to Oberlin.

I had saved enough money for Tuition, but still needed to pay for Room and Board and Entertainment. So, when I studied a subject, especially Math, I would tutor some classmates the way I had tutored Freddie. It actually helped me learn it better for myself too.

SPIKE Pedagogy, which always works if a student wants to learn for either an extrinsic or intrinsic reason. Good Grades are extrinsic motivators. Proper Content will often become an intrinsic motivator.

These students could repay me in various ways for my help. Sometimes they tutored me. Sometimes it was just friendship. Sometimes they would pay money.

I became pretty well known among my fellow students and also with some of the professors. In my sophomore year at Oberlin, I was made a Math Problem Session Instructor, a well paid position. Usually only Juniors or Seniors could do this.

My job was to meet with the students enrolled in a certain Math class and teach them the Math their Professor was not succeeding in teaching them in the Classroom, usually with Lectures.

Sort of like Freddie and Miss O'Hair.

I would always use the SPIKE approach with each individual student since students could come and go and only a few were there at any one time. I could tutor one and then while that student was working on a problem I could tutor another, etc. It was much more effective than trying to teach more than one at a time.

Sometimes I could teach a few students at the same time using the Socratic Technique if they did have the same Prerequisite knowledge and similar abilities.

The Socratic Technique (my definition) is simply asking the students a series of questions until they understand a concept. This can take many different branches depending on the Student. Rarely are any two Students the same.

This is a difficult technique for some teachers to do. It is much more difficult than giving a Lecture where you just present material in a linear way at a pace that is comfortable for you, the teacher.

I rarely ever had a Teacher use the Socratic Technique in a Math Class I was a student in. But if I went to their Office, they could sort of do this.

I never lectured to a Math Class. I always used the Socratic Technique.

Does it work for all students in a large class?

Sadly, NO.

You have to go at some pace in order to complete the material dictated by your boss. Some students just can't keep up. Other students are bored because you are going too slow for them. My two complaints or criticisms in a Class when I was a Teacher were: Too Fast for some and Too Slow for others.

This was Never a complaint with a one-on-one SPIKE Tutorial Student. Unfortunately, many students can't find or afford a good SPIKE Tutor.

I made a pretty good living Tutoring Math and as a Problem Session Instructor at Oberlin. Probably about $25/Hr to $50/Hr in 2022 Dollars.

BUT that was just the beginning!

Math can have many uses, and open up many other doors for success.

I call Math the 'Open Sesame' for many Opportunities!

This really started at Oberlin, and then several years later, Big Time.

Chapter 16: Math Applications – Hardest Courses, Poker, Bridge

"What good is Math?," is a frequently asked question by a Math Student. Or, "What's Math good for?" or, "Why do I have to learn Math?"

All good questions. What would your answer be to a student?

I like to respond Socratically.

What good is a language like English? See how the student answers. Usually the student finally figures out that life would be very difficult without becoming somewhat Literate.

It might take a whole series of questions before the student came to that realization. I never said the Socratic Technique is easy for a teacher. But it is Fun and Satisfying for a Teacher who masters it!

It is rarely the same for two different students. The Goal is to get the student to finally give good answers to a series of questions resulting in learning the Topic being taught.

So you start with a Question that could lead to a series of Questions resulting in the understanding of a Topic. If the student cannot answer that first Question, you begin asking a sequence of other more basic Questions that will lead to the Student being able to answer the Question you started with.

Then you move on down the set of Questions resulting in the Student understanding the Topic. I didn't say it was

easy for a Tutor or Teacher to do. But I don't know any better way to do it for most students.
Unfortunately, in my experience, most Teachers really can't do it. Their excuse, of course, is that no one can do it with a group of students with varying backgrounds and abilities. Which is true.

So Lectures are easy! And most students Fail!

Certainly, No SPIKE Pedagogy.

OK. Now about Math and the Socratic approach.

Did you know that Math is a Language? Then with a series of Q & A's, the student realizes that Math is a Language. Symbols and Concepts are like words in a natural Language. Indeed, you can express ideas and concepts in the Math Language you cannot express in any Natural Language.

And Math is a Universal Language. Math Concepts are the same in China or Russia or Africa as they are in the USA. Indeed, I call Math "God's Language."

More importantly, Math can be used to find the answers to many questions you might have that cannot be answered with any natural language.

Our ancestors discovered this at the beginning of Civilization. Just counting things in the original economies could only be done with Math.

Then what we call Science empowered Sapiens to understand all sorts of natural phenomena that we could not understand in any other way. Math is the language of Science.

Try to describe accurately the trajectory of a bullet fired at a 42 degree angle with some initial velocity using English or any other natural language.

It is possible, and fairly easy, to do so with the Math Language embedded in some Science Model. It's a Parabolic Curve! The Graph of a Quadratic Polynomial Function. You can predict where the bullet will land with Calculus! Try that with any Natural Language.

Engineering is an application of Science that can only exist with Math to express Concepts and Solve Problems. An engineer sets up a Math Model and solves it to design something. Or, the engineer experiments with various prototypes and then tries to set up a Math Model to predict all of its behaviors, particularly those that could lead to some disaster. Technology is simply various applications of Science and Engineering, and most technologies require Math to understand and apply them.

So, we now have an acronym, STEM

Science **T**echnology **E**ngineering **M**ath.

It probably should be MSET since Math comes before there will be any Science, which comes before Engineering, which comes before Technologies.

Do you get it? **M S E T**

OK. Back to Oberlin.

I learned that most students of a Science subject were often defeated by the required Math. So, as a Math major, I would sign up for the most difficult Science course in a science field and they would let me in even if I didn't have the normal Prerequisite Science courses.

Physical Chemistry and Theoretical Physics were the most fun. I found them easy to master thanks to my Math understanding where those fields' majors found them difficult, because they lacked the appropriate Math knowledge.

Fun, but no Money in that!

Poker. Aha. Money. Just a little basic Math and some Psychology and you can become a good Poker Player. Fun for me. And some Money. Unfortunately, Playing Good Poker is VERY BORING!

Bridge is similar. Duplicate Bridge was fun for tournaments, but Rubber Bridge was the money maker. Less Boring, BUT requires a Good Partner, which can be difficult to find!

I was able to earn an A in Oberlin's most difficult Economics course with Professor Tufts, thanks to Math that most of the other students lacked. English and History, not so much. But even there, sometimes Math helped, if for no other reason than Logic or Demographics. Math helps a lot with Geography too.

So between Tutoring, Problem Session Instructor, Poker and Bridge I was able to graduate from Oberlin with No Debt.

Students at Oberlin studied many fields. English was my second major. My senior year, in an advanced English Seminar, I met a local high school English Teacher, who because of my reputation was able to get me a job as a Math Teacher at a local High School 20 miles west of Oberlin after I graduated.

Wow. What an experience... Western Reserve High School. My “Math Education” Education really accelerated!

Best Teaching Year of my Life... and Worst!

The first of MANY Post-Oberlin Adventures.

For me, my Parents were right. College was an ‘Open Sesame’ for many Successes and Failures.

Chapter 17: My Experiences at Western Reserve High School

My four years at Oberlin College were just a wonderful series of Adventures. I could write a long book about them, maybe someday.

I had No Plans for the future after I graduated in May of 1960. No ambitions, except to continue to have Adventures.

Well, the next one came at Western Reserve High School, in Wakeman, Ohio, 20 miles west of Oberlin on US Route 20, a small rural community High School with a couple of hundred students Grades 9 – 12. About the size of GHS.

Turns out their Math Teacher, Mr. Jenks, was retiring after 15 years, at age 87.

One of their English Teachers, Mrs. Drennon, was attending an English Seminar at Oberlin, and got to know me. She knew that I was also a Math Major and had quite a reputation as a Math Tutor. She asked me if I would like to teach High School Math. Of course… Yes!

So, I met the Principal, Mr. Runyan, and he asked what Math Education Classes I had taken. Answer, None. Oberlin had None. Indeed, what were they?

"Well, I can get you an Ohio State waiver to teach Math for one year," he said.

He offered me the job for $4,000 for a nine month year. That was about $40,000 in 2022 Dollars and double what I had been living on for several years.

Great! Wow, now I will be a real Math Teacher. Not just at Tutor.

“You will be our only Math Teacher. You will teach two Classes of Algebra 1 for 9th Grade, two Classes of Geometry for 10th Grade, one Class of Algebra 2 for 11th Grade and one Class of Seniors Honor Math for the 12th Grade,” explained Mr. Runyan.

Hmm. 6 Math Classes. 50 minute periods with 2 minutes between periods, and a 30 minute Lunch Hour..

“Oh Yes, we have 7 Classes in a day. Could you teach Physics or Chemistry for the 7th Class?,” asked Mr. Runyan.

“Sure! Which one?, “ asked little ole me.

“Well, our Superintendent can teach Physics, so you can teach Chemistry,” he said.

Great. That is a whole nother story. Let's stick to Math for this book.

“And, you can have ten “sick days” without a doctor's excuse if you are sick, with no loss of pay, said Mr. Runyon. I think I used all of them for some Fridays off. After all, I still had Poker and Bridge to play!

“No extracurricular duties, but good to attend some ball games and dances.” he said.

Fine.

“Oh yes, Discipline.

We paddle students when necessary, but you need another teacher as a witness and to use the paddle I, the Principal, provides,” he said.

“OK,” little ole me.

“And, I do not smile before Thanksgiving in front of any students,” he concluded.

Actually, Mr. Runyan was a pretty good Principal. He counseled me on how to maintain discipline, especially in Home Room, which was the first class of the day.

The students all would have math textbooks already chosen by the school.

Wow! The Fun began.

As you know, I used the Socratic Technique to teach Math.

I soon found out that teaching a Class of 15- 20 Students was much more difficult than Tutoring One-on-One. MUCH MORE!

In a typical Class at a good school where most students have a good home life, in a class of 20 students, if you teach to the middle 10-12 students in terms of ability to learn math and background, the lower 4 or 5 won't be able to keep up, and the upper 4 or 5 will be bored and lacking challenge. The middle 10 – 12 will do just fine.

I can't imagine what it's like in a bad school with huge discipline problems and a lot of students with difficult home lives who all have a large variance in math backgrounds. I'm told that's common in many Public Schools today in 2022.

It was very frustrating for me to not be able to properly tutor the slower and more poorly prepared students. Sometimes it was lower ability and more often it was lacking in pre-requisite knowledge.

And, to not go fast enough for the best students. Maybe, that's even worse. In a sense, you are really "cheating" them of a better Math Education.

I learned something disturbing after about two weeks. I gave my Algebra 1 students a little test and most of them did pretty good.

My 11th grade Algebra 2 students had been struggling and seemed to be unprepared. So I gave them all the same Algebra 1 test. They did Much More Poorly than my 9th Grader Freshman.

Now I knew it was not their ability. I had the 11th Graders in Chemistry and they were doing fine. They simply did not know much Algebra 1 and I found out that they had not been taught by Mr. Jenks. Surprise?

So the Juniors had to first learn Algebra 1 and it took about a Semester before they could then learn some Algebra 2.

My Seniors did just fine because I was teaching them some modern math like Set Theory and Logic, and some Trigonometry and Analytical Geometry too.

They were really all pretty good students. I have found that is typical for students who come from good homes. Sure, some are slower, but even they can learn with SPIKE Pedagogy (See Chapter 26).

Unfortunately, I learned it is impossible to apply SPIKE Pedagogy to all students in a Group.
There is just not enough time for each student, especially the slower ones.

One of the Seniors, Jim Fletcher, did so well I was able to get him accepted to Oberlin College, just like Dr. Gass had done for me.

Many teachers paddled students who disobeyed the rules. I hated this and always found ways to punish or motivate students in other ways.

However, one day after my Senior Class there was a thumbtack in my seat I sat on after I came back from my two minute break we had between classes. "Ouch!"

I told the next Class that I would punish the entire Senior Class for this.

After school, our star athlete, Jerry Maul, came to me and confessed he had put it there and was sorry. I thanked him for his confession. Then he told me he had heard that I was going to punish his entire class.

I said, "Yes, since they watched you do it they are sort of accomplices."

"Please NO, don't do that! It was not their fault. It was all mine. Please just punish me. I can't play basketball tonight worrying about this. Please," pleaded this Star of our Athletes and Captain of the Basketball Team.

Maul was a very strong, well behaved young man. A good student. Hero to many other students.

"How? What do you want me to do?," I asked in a quandary of what to do.

"Well, why don't you paddle me like the other teachers paddle rule breakers. I can take it. Please," replied Maul confidently.

So after some conversation I agreed to do it, but his classmates would have to watch and think about their lack of actions and responsibilities too.

He thanked me and played ball that Friday night. Wow, what a guy!

I went to the game and, as always, talked to a lot of students. I really loved this student body. They were great kids. In fact, they taught me how to dance at a dance party!

The president of the Senior Class, Jim Fletcher, who was my best Senior Math student that I later got accepted to Oberlin, was talking to me in a group of students.

He said he had heard I was going to Paddle Jerry Maul Monday.

“Yes,” I said. “That was our agreement so I wouldn't punish the rest of the Class which includes you. Maul will take your punishment for you,” I replied.

“I'll just bet you won't have the nerve to paddle Maul,” Fletcher retorted in front of this group of students.

“Well, the bet is on, and the bet is that I will paddle YOU too right after I paddle Maul! So, You too, can accept responsibility for the rest of the Class along with Maul. After all, you are the Senior Class President. You could have stopped Maul from putting the thumbtack in my seat,” I said.

Jim turned pale and was then as quiet as I had ever seen him. Great kid.

Come Monday, Jim and Jerry, both got paddled by me with Mr. Runyan watching.

I gave each a firm painful swat, but certainly not enough to really hurt them. Great kids. Loved them both! Along with the other students.

Well, the school year ended and I had to leave for more unknown Adventures and many of the students came to say goodbye on my last day.

I knew they were going to have a weak Math Teacher going forward, but what could I do? I hugged them goodbye. Very sad.

My best year of teaching math classes ever.

I wished I could teach them and all other students all over the USA math.

I wished I could not only teach them, but Tutor them so they could All learn the Math they needed for their futures. SPIKE Pedagogy. Proper Content.

But it was Not Possible at that point in time with the technologies that existed then.

Now, in this 21st Century, I CAN do it, and am doing it!

And perhaps You, the reader, can participate in this wonderful process.

Read On in Part V to learn how!

But first, one more story in this Part III.

Chapter 18: DePauw: Set Theory and Logic – Real No. System - Infinity

It was 1961, and I was Back Home again in Greencastle.

What was going to be my next Adventure? I was considering becoming an Actuary since Math was involved.

I was talking to Dr. Gass, as I usually did when I was in Greencastle, and he asked me what my plans were. I said, "No real plans yet."

He asked me if I would like to teach Second Semester at DePauw next year while he went on a Sabbatical Leave.

What? DePauw? At age 22?. Do I know enough Math? Dr. Gass assured me that I had learned some Math at Oberlin that none of the DePauw professors knew, and I could teach an advanced Math Theory Class.

Then he also asked me about going to Math Graduate School. I had not considered that at all. He suggested I go to Indiana University Math Graduate School since it was one of the best in the country and to try it out in the first semester.

So in September, I enrolled as a Grad Student at I.U.'s Math Dept in Swain Hall. It was the first time I had ever visited Bloomington, 50 miles from Greencastle. Another Adventure. Two in one year!

I'll tell the I.U. story in another Essay or book someday. Let me just say the first year at I.U. was interesting and enlightening in several ways.

During the second semester, I taught Math at DePauw to their Senior Math Majors. I had been very overwhelmed by many of the theoretical Math Concepts at Oberlin, and now I.U. Irrational Numbers, Different Sizes of Infinite Sets, Godel's Incompleteness Theorem. Most of these topics are of interest only to theoretical mathematicians, not to Science, Engineering or Technology students. So the students and I discussed them and tried to learn more together.

Wow, teaching at DePauw at age 22. My GHS folks couldn't believe it. How could a kid who didn't do well in Algebra at GHS do this?

I hate written timed exams. So I gave all my students an Oral Exam. Turns out I can find out in less than an Hour just what a student has learned and understood in a class I am teaching the student.

In Math, for any given Topic, you either understand it or you don't. There is no such thing as a C in Math on any specific single Topic.

Of course, if you teach 10 topics, I guess you can give a C if a student understands 5 of them. But, which 5? And, what about the 5 topics the student does not understand?

Usually, it is the most important 5 topics for that course. The next course assumes the student knows all 10 topics. Rarely does a student have all of the Prerequisite knowledge needed for any given course.

So, after a C in one course, the student probably will Fail the next course.

That leads to the Classroom's Unsolvable Problem!

That will be discussed in depth to kick off Part IV.

Part IV:
Identifying the
Public School
“Math Education Problem”

Chapter 19: The Classroom's Unsolvable Problem, for Both Teacher and Student

As a student, I had always observed that many of my fellow students simply did not understand whatever the teacher was trying to teach them. Often, I didn't either!

Indeed, as a Tutor at Oberlin, I was always amazed at how many students just did not learn the math from their teacher in the Classroom. I blamed the teacher, but it helped me earn a good living.

Then, when I taught at Western Reserve High School and DePauw, both high school and college, I realized that many students simply lack the prerequisite knowledge to understand the current topic being taught. I also learned that some students could not absorb and understand a new topic as fast as some other students, and could not keep up with the pace of the class.

There is very little a Teacher can do to overcome this problem. There simply are not enough time and resources in a typical Math Class to help the slower performing students. Their only hope is to get a good Tutor. You cannot deliver SPIKE Pedagogy in a Classroom to all students. See Chapter 26 for a detailed description of SPIKE Pedagogy.

And the better prepared students suffer too. They become bored and their time is wasted. This is true in virtually all Math Courses taught in both High School and College.

However, in College with advanced courses for Math Majors, this is not so much a problem since the students all have similar abilities and backgrounds.

If you teach Math with the Socratic Technique (Review Chapter 16) in the Classroom, most students will learn well. In my advanced Classes like Topology, I would tell the Class that there would be no Curve. They could all get A's and they could all get F's. Probably a few D's, B's and rarely a C. Either you understand what I am teaching you or you don't. You'll probably get an A or B or D or F depending on how hard you work. C's are rare.

I would have Office Hours where the students could come for one-on-one Tutorial help for a topic when they needed it. This really helped ambitious students who simply could not keep up the pace in the Classroom.

When I taught advanced theoretical Math as Professor for seven years after I earned my Ph.D., I had students who usually got A's or B's. My students knew that I did not grade on a Curve.

I gave Oral Exams so we both knew what the student knew or didn't know. This is not possible in most High School Math Classes unless you somehow segregate the students according to their abilities and current knowledge.

This has Nothing to do with Race, Religion or Gender. Only what Math they already know, their Ability to learn new Math and how Hard they would Work. By the way, Hard Work is often even more important than so-called Ability.

SPIKE Pedagogy takes all of this into account. A Math Program that practices SPIKE Pedagogy will have very few failed students and mostly successful students. See Chapter 26 for a detailed description.

The Proper Content does vary depending on the Student's other interests and future work plans.

I will discuss that extensively in Part V.

Chapter 20: Theoretical Math and the Student's Psychology

This Chapter is intended to demonstrate and explain one way our current Public School Math Programs harm many Student's Psychology.

Just read it superficially and get an idea of just how and why things work in today's Math Ed Programs. You can jump to the end for Conclusions. But, hopefully you'll "enjoy" a quick superficial reading, unless you happen to be a mathematician, which should bring back some memories.

Mathematics consists of what are called Axiomatic Systems. You start with a set of Undefined Terms. Then you create a set of Axioms, which are Statements about the Relationships of these Undefined Terms to each other that are assumed to be True. Then you prove a set of Theorems which are Statements about the Undefined Terms you can Prove in a finite set of steps from the Axioms.

So, as a great Mathematician once said, "A Mathematician is someone who knows a great deal about Nothing." However, we sometimes give the Undefined Terms what are called "Ostensive Definitions" so we can apply the Math to some real things, or something else we can imagine and understand.

In Euclidean Geometry, Point and Straight Line might be Undefined Terms. We imagine, or Ostensively Define, a Point as a position is space or a dot on a piece of paper. A Straight Line is the set of points connecting two points with the least distance between them, if you have what is called a metric, or distance measure. Don't worry, Theoretical Math can get quite complicated.

For example, we believe in Numbers starting with Counting or Natural Numbers. This Set of Numbers is treated Axiomatically with Peano's Axiom System. (No, that's not one of my misspellings. Peano was the name of a mathematician who created this Axiom System.)

It goes on from there. Fractions are called Rational Numbers, like 17/23 or A/B when A and B are Natural Numbers and B is not equal to zero.

Can't divide by zero, that's a No-No.

Fractions can be represented by an infinite decimal expansion if you understand the decimal number system. 1/3 = .333333. . . meaning this goes on forever or to infinity, whatever that means. .33333 is just an approximation of 1/3.

All Rational Numbers have decimal expansions which have repeating segments, forever. 1/7 = .142857142857142857 . . . You see what repeats? 142857

What about 137/253 ? = .54150197628458498023715 ...repeats this forever...

BTW, it took me less than one minute to find this out! Modern Math Tool!

What about numbers that have a decimal expansion that never has any repeating segment? These are called Irrational Numbers.

Example. Square Root of 2. The Pythagoreans discovered this a long time ago.

They did not believe any such numbers should exist and they sort of abandoned numbers systems in favor of Geometry. In a sense, they are right. There are No Irrational Numbers in the Real World.

All things are measured with Rational Numbers, always with an error +/- term.

But, Mathematicians love the set of "Real Numbers" which, to a Mathematician, is both Rational and Irrational Numbers. Indeed, there are an Infinite Number of Natural Numbers and also an Infinite Number of Rational Numbers. These Infinite Sets are in some sense the same size. Infinity.

Then in the late 19th Century using an Axiomatic System, the mathematician Cantor proved that the Infinite set of Irrational Numbers is larger, in a well-defined sense, than the Infinite set of Rational Numbers. Some mathematicians thought Cantor was crazy, but his ideas won out in the end in the 20th Century.

Indeed, consider all the Real Numbers between 0 and 1 on the Number Line.

Between any two Rational Numbers is an Irrational Number, indeed, an Infinite number of Irrational Numbers. Also, between any two Irrational Numbers there are an Infinite number of Rational Numbers.

So what is the Probability you will randomly select a Rational Number if you randomly pick a Real Number between 0 and 1? You might guess ½. Seems like a good Guess. Could you "prove it"?

Equal chance of picking a Rational or Irrational Number?

Wrong! The correct answer is Probability = 0.

The Measure of the set of Real numbers between 0 and 1 is 1.

The Measure of the set of Rational numbers between 0 and 1 is 0.

What is the Measure of a set of Irrational numbers between 0 and 1?

You might have guessed it: 1.

The Probability of selecting an Irrational Number randomly is, "drum roll," 1!

This is theoretical Math. No relationship to the real world we live in.

Most Math students could care less about this type of Math. It is very difficult to understand and accept. I found this to be true at Oberlin by my Sophomore year.

No one at DePauw knew it. Dr. Gass was amazed. "Are you sure?," he asked.

Well, Theoretical Math gets Much Worse than this!

One of the Axioms of Set Theory that all modern mathematicians accept is called the Axiom of Choice. Many of the most important Theorems in our modern Axiomatic Systems depend on this Axiom of Choice. Some of the things you can then prove from this Axiom of Choice defy all intuition, mine at least, and could never happen in the real world.

It has an equivalent Axiom called: Zorn's Lemma, named after Max Zorn.

I was a friend of Max Zorn when he was a professor at I.U. when I was a graduate student there in 1962-66. We were about three offices apart. I had many interesting discussions with Max.

Wild and Crazy guy! But Brilliant and wonderful.

I pretty much concluded that all theoretical mathematicians are Crazy!.

"Crazy" in that they don't think like 99.99% of other Sapiens.

It turns out that these Wild and Crazy guys had a big influence on the Math Educators who created the modern Math Curriculum used in Public Schools for all of our students starting in the mid-20th Century.

For example, we teach 9th Grade students that the Square Root of 2 is Irrational. So is the square root of 12.1 But the square root of 1.21 is Rational.

Who cares? You tell me.

No Scientist or Engineer I ever knew cares. 1.4142 is close enough most of the time. You never will need more than 20 decimals for any calculation.

OK. The Square Root of 2 is approximately:

1.414213562373095048801688724209698078 ... and don't you forget it! Haa!

With a Modern Math Tool, I could have told you its approximation to a thousand decimal places or more. No Scientist or Engineer or Technician cares.

Conclusions

There are all sorts of Math Theory in many of our current Math Textbooks used by our Public Schools. I just gave you a small sample. Difficult and irrelevant to most students. This sometimes causes students to dislike Math and often fail their Math Courses. Depends a lot on the specific Math Teacher.

What does it do to a student's Psychology to be forced to study and learn things that are never going to be of any use to the student and are very difficult to learn? You tell me.

Did You find the above discussion Easy?

I can tell you my answer. It is very damaging to many student's psychology regarding Math.

There are exceptions, of course, like the rare future Mathematician who needs a TOTALLY different type of Math Education.

This inappropriate Theory will be avoided when Proper Content is selected like I have tried to do in my Six Tier Program.

Then, there is the Issue of Math Tools. The situation is even much worse there.

I think you will be amazed by what you will learn in Chapter 21 and then you will understand this. I know I was amazed when I learned about these new 21st Century Math Tools and still am!

These new Tools totally change how we Learn and Apply Math, as well as solve Math Problems. These Math Tools Revolutionize 21st Century Math Education.

Public School's Math Programs haven't caught up... Yet.

Chapter 21: Math Tools and Student's Future Success

Tools Tools Tools Tools Tools . . . .

Our Lives and our Civilization depends on the Tools we have and can use. Tools to start and control Fire! The First Tool for Civilization?

Hunting Tools - Transportation Tools - Building Tools - Science Tools.

That's the Sapien Story isn't it?

David had a more modern Tool than Goliath did and what happened? Goliath was actually the "underdog" in the fight with David, who won the battle thanks to his superior tool, the Sling. It was like using a Gun to fight a person with only a Sword. Goliath never had a chance.

Language was the first Tool we needed to communicate. It probably came before Fire or Hunting Tools. Math is a Language. Without this Tool there would be no Sapien Civilization.

A professor once told us that the first number system was:

1 , 2, 3, Heap. (from Cavemen?)
Later came 1,2,3,4,5, . . . Infinity (Greeks and other ancients)

Then came 1,2,3,4,5, . . . Ao, 2^Ao = c, 2^c, 2^(2^c), . . . (Math in the late 1800's.)

Ao is Aleph Null, the Infinite Number Size of the Set of Natural Numbers and Set of Rational Numbers

Size of the Set of Irrational Numbers and Set of Real Numbers is c = 2^Ao which is a larger Infinity Size.

So, lots of Sizes of Infinity.

Indeed, an Infinite number of Infinity Sizes.

OK. Math Tools

The simplest Math Tool might be the + Tool.

2 + 5 = 7 "=" means the same size or same number of elements in a set.

Then came the x Tool 4x3 = (3+3+3+3) = (4+4+4) = 12 = 10 + 2

This is what we teach our young children today in the USA.

To start, the easiest thing might be to memorize the + and X tables 1, . . ., 12

Then there are Algorithm Tools. What?

The ways or processes we teach students to add large numbers or multiply large numbers or do many other things with Math Concepts.

Then comes / or division. 15/3= 5 11/4 = 2.75

Then squares. 3 squared = 3^2 = 3x3 = 9

Then comes square roots. Square root of 9 = 9^(1/2) = 3

Then Powers. 3^5 = 3x3x3x3x3 = 243

But, what about 1.2^6.7 = 3.39?

Impossible without a more powerful tool.

So we invented Logarithms, the Inverse of Exponentials. This calculation was difficult, but possible, when using a Manual Tool called Logarithm.

MUCH Quicker and easier with the y^x Key on a Scientific Calculator like the TI-30Xa. 10 Seconds compared to 10 Minutes or longer.

Our ancestors created a plethora of Wonderful Manual Tools. Logarithms Tables and their Inverses Exponents. Trigonometry Tables. Slide Rules or Slipsticks, long linear or round. With these Tables/Tools we could solve very difficult Number and Geometry Problems. These Tools were all Manual. Both Difficult to Learn and Master.

But Powerful in solving certain Math Problems that came up in Technical Situations, Economic Situations and War Situations, etc.

Compare them to the Manual Carpentry Tools of the 1700's and 1800's. That's when many of the Math Manual Tools were invented or created. Wonderful, and empowered Sapiens to build wonderful structures. Our ancestors were geniuses and we owe our modern Civilization to them.

Question: Would you want to use these Manual Tools today to build a house?

All of the Manual Tools from the 1700's and 1800's for doing all sorts of arithmetic calculations are now superseded by a $10 Scientific Calculator.

How many Employers will pay you to do arithmetic calculations with the old Manual Tools? None, I know of.

Why? Very time consuming and Error Prone. IF you can even do it. These Manual Tools were HARD TO LEARN and MASTER.

For Example: What is (1.2 + 5/9)^5? Do it manually.

This is possible. How long and hard was it to do it?

Answer: 16.68 I just did it in 20 seconds with a TI-30Xa Scientific Calculator and I'm an old slow guy. My students are much faster.

SO, I believe that the first thing we should do is teach a student the Concepts of + and – and x and / with simple easy problems so they understand the Concepts. Then teach them to use the modern tool of a Scientific Calculator.

Once they learn one Calculator, it will be easy to learn another one. Ultimately, they probably will use one on their Smart phone.

To learn how to make dealing with Fractions very easy, or to Master the TI-30Xa Scientific Calculator, go to the Special Offers on p. 163.

There you will learn about two Online Courses to do either of these things very cost effectively in terms of both time and money with No Risk.

Once a student Masters the TI-30Xa Scientific Calculator, which takes most post-elementary students only a week or two, then the student is ready to learn:

PRACTICAL - Algebra – Geometry - Trigonometry in about one semester. That is if you use SPIKE Pedagogy to teach it. (See Chapter 26) And… Proper Content. This is Tiers 1 and 2 in Triad Math's Six Tier Math Program.

Now the student is ready to learn many Technical Subjects and Processes, and enter an Apprentice Program or the Military.

The Student who understands Tiers 1 and 2 will know more math than 95% of USA Public High School graduates. That is how bad our Public Schools have FAILED our Students.

In Less than one year around the 8th or 9th Grade, all students should learn:

PRACTICAL -
Algebra – Geometry – Trigonometry

To see how this is possible, go to the Special Offers on p. 163.

Then the Student will have a great Foundation to continue learning more advanced Math for Consumer things and then STEM subjects. Also, the Student will have "Learned to Learn." The Student will have great Self-Confidence and Self-Esteem.

Are any Public School Math Programs doing this?

That is why this book's Title is: *Public School Math is Destroying the USA*.

I consider it a "Sin" not to teach our Students Proper Content like this.

Furthermore, for future College Students and STEM students, the Fun has just begun!

An Incredible 21st Century Math Tool (2009), named *Wolfram Alpha*, does for ALL PreCalculus Subjects, Calculus, Differential Equations and All STEM Math MUCH MORE than the Scientific Calculator did for Arithmetic.

Amazing. *Wolfram Alpha* still seems like a Miracle Tool to me.

Now, a future STEM student can learn STEM Math in a couple of years.

The difficulty level of Calculus has gone from an 8 down to a 2 on a scale of 10 with 10 being the most difficult..

Integral Calculus and the application of the Fundamental Theorem of Calculus, which requires finding the Antiderivative of a Function, has gone from a Math Barrier of difficulty 7 or 8 to a very easy Path with a difficulty of 2. This is why I teach Wolfram Alpha extensively in Tiers 4, 5 and 6.

Does ANY Public School teach such a modern 21st Century Math Tool? Does any Public School teach STEM students Differential Equations, the workhorses of STEM?

... *Destroying the USA?* What do You think?

In Tier 3, I still just use the Calculator since you cannot use Wolfram Alpha on the SAT or ACT. Those, in My Opinion, are horrible tests our Public Schools teach for, and they really prove very little. Maybe they prove which students can afford Tutors or extra help to pass them.

What's Your Opinion?

Look at the Six Tier Syllabus in the Appendix to see one Math Program that teaches a Student Proper Content utilizing SPIKE Pedagogy.

Compare any other Math Program to it.

Eliminate inappropriate Theoretical Concepts, Obsolete Manual Tools and Deliver your math lessons using SPIKE Pedagogy.

That will prove to be very successful 21st Century Math that virtually all Students will benefit from.

The Proof is in the Pudding, and any family can start all of their Students down the Six Tier Path for only $29/Month.

Go to the Special Offers on p. 163 to learn how.

Very little to lose, and much to gain.

A Convex Option according to Nassim Taleb.

This will be explained in the *Dr. Del's "Fun with Math" Club* you will be an automatic member of.

The whole Mission is to demonstrate what Public Schools COULD DO!

. . .Vastly Improve the USA!

Chapter 22: The Horrible Bell Curve

Suppose you study a Math Subject, A, with 20 Topics. A Topic can be a Concept or a Tool you need to learn in Subject A.

Suppose you then study a Math Subject, B, which assumes you already know the 20 Topics from Subject A. Subject B uses them as it teaches you its new Topics.

Suppose you learned 10 of the Topics in Subject A and not the other 10 Topics. How well prepared are you for Subject B? What do you think your Grade for Subject A should be since you learned 10 of the Topics but not the other 10?

Well, if you were Graded on a Bell Curve, the answer would probably be a C.

How well do you think you will do in Subject B?

So, what is this "Bell Curve" thing?

Why did you get the Grade C in Subject A?

Well, you were tested on the 20 Topics in Subject A, and if you learned 18 – 20 of them you got an "A", 14 – 17 a "B", 8 -13 a "C", 3 – 8 a "D" and 0 to 2 an "F."

Furthermore the tests on Subject A are constructed so that in a typical class the Grades will be distributed whereby the percentages of students in the class getting the Grades will be like 10% A, 20% B, 40% C, 20% D and 10% F. That is what is called a "Bell Curve" or "Normal Curve" of Grades. If you plot it on Graph Paper the Graph will look like a Bell Curve.

Now it is possible to construct the Tests so that the Grade Results can fit a Bell Curve. There is a whole "Science" on how to do this.

How many of the Students do you think are ready to advance to the next Class?

Some Math Educators have been taught that the right way to evaluate a Class's Performance via the Grades is to achieve a Bell Curve. Statisticians love a Normal Distribution, or Guassian Distribution, or Bell Curve, all names for the same thing.

In a Normal Distribution Bell Curve, you have an Average or Mean Value and then a Standard Deviation, which measures how many Grades from the Mean.

Only one Problem with this!

With a Math Topic you either understand it or you don't. A Math Problem has one correct solution and all others are wrong. If you have a Math Program, either a Student masters it and is ready to use it in the future or not.

It is my Belief that a Student either Passes a Class by Mastering virtually all of the Topics, or Fails the Class and should take it again until all the Topics are Mastered. Then they can go on with the Next Class that requires this Class as a Prerequisite.

In fact, Life is usually more complicated than this. Often the 20 Topics are inter-related and somewhat depend on each other. So, you really can't understand some of them without understanding the others.

It is actually artificial to divide the Subject into a set of independent topics so you can learn some of them and not others. If that is the case, then you should have two Subjects or Classes.

My experience has been that a Math Student who has a series of C's in a series of Math Subjects probably doesn't know any of them adequately.

It's sort of like riding a bike. You must do several different things successfully to ride a bike, and either you can do them all and ride the bike, or you can't ride the bike properly.

Suppose you can pedal and go forward but haven't learned to brake properly? What will the result be?

Probably okay if you never have to brake properly, but a disaster if and when you have to brake properly.

OK, I will test you in a Bike Riding Class. What grade should I give you? How about a C since you learned five out of the ten things you must master to ride a bike. Congrats, You Passed. Or maybe a B, even though you still can't brake properly... an even better Grade.

Math is sort of like that in any given Subject. You understand the Subject and all its various Topics or you don't.

When you Grade on a Bell Curve, you are deceiving the Student about the Student's abilities. Unfortunately, that is what many of our Math Educators teach and practice and teach the Teachers to practice. If you plot the Grades in a Curve, it looks like a large Bell Curve with one big hump in the middle (Grade C) and trails off in both directions..

But, if you Grade a Math Student properly it will be what Staticians call a Bi-Modal Distribution. A Bunch or Curve of “F's” and a Bunch or Curve of “A's.” There may be some “D's” for students who really tried hard and failed, and “B's” for Students who don't do well on timed tests and make careless mistakes, but essentially understand the Topics. But very few Grade “C's”.

BTW, That is how Math Courses are graded in Graduate School. You mastered the Subject or you Didn't. Pass or Fail.

Now, to give the Math Educators some slack, they have had a difficult problem historically. In most large Math Classes with a wide variety of Students with different abilities and backgrounds, it is very difficult to teach the Class in a way that is optimal for each student. Slower or less well-prepared Students will need to go at a much slower pace with more feedback than Faster or better prepared Students. This is a horrible problem for a Teacher with a large Class.

The "Solution" created by Math Educators back in the 20th Century was the Bell Curve. What else could they do given the limited resources? Unfortunately, it was a disaster for 90% of the Math Students. Those who got less than a B would get behind and NEVER catch up.

All it took was ONE bad Math Class, like I had in Algebra 1 my Freshman year, as you learned in Chapter 8. I was Very Lucky or Fortunate as I explained in Part I.

OK, Math Grades!

Well, the Grades for a group of Math Students in a Math Class usually should be Bi-Modal. Either you Understand the Math in a Subject or you don't Understand it. You get a B or A if you understand it and a F or D if you don't understand it. There is not really any place for a C. So, the Distribution is Bi-Modal.

Of course, the concepts of Mean or Average and Standard Deviation don't mean anything and don't apply. Those are only useful with a Bell Curve Distribution.

Most Math Educators disagree with this, at least that has been my experience.

Indeed, it is possible to construct Math tests, like the SAT, where the Distribution of Scores is Normal, or Bell Curve.

But, in a given Math Subject, most students will either essentially understand all of the Concepts and be able to do the Problems or not.

I have never met a student who got a C in Calculus that had a meaningful or useful understanding of Calculus. Same for other Math subjects.

However, in a typical Math Class of a Group of Students, the teacher will structure the tests so that the scores are sort of a Normal Distribution, or Bell Curve. They've been taught to do this by their Math Educators.

However, an Honest Test will result in a Bi-Modal Curve. Most Students will either understand the subject really well or not understand it well at all.

Most honest Grades should be “F” or “A”.

A “D” means a student tried hard but essentially failed to learn the subject. A “B” will mean a student essentially understands the Concepts but probably makes a lot of careless errors.

THIS is a very controversial position!

Math Educators I have known disagree with THIS! Heck, it got me fired in 1968 from a Math Education Dept. at ISU. In the next Chapter 23, you'll see how I learned this the hard way.

Math Educators work harder to make the Students grades fit a Bell Curve by how the tests are constructed than to figure out how to make all students succeed.

In most Public School Math classes, truth be known, 20% should get a B or A, and 80% should get a F or D.

This is why I say it is Destroying the USA.

In my advanced classes it would be 90% get a B or A, and 10% an F or D.

This was because my students knew in advance that the grade they earned would be based on what they were supposed to learn, and NOT on any Curve.

Indeed, in many Math Classes, the Teacher simply graded on the Curve no matter what the students learned, and there were enough things to memorize without understanding much of anything to get a Bell Curve. Even the A students didn't learn much!

Sometimes a good student would learn much more and score a much higher score. This was called “ruining the curve” and deplored by the other students.
Smarty Pants ruined the curve. Not a way to be popular!

With my Six Tier Program, a Student cannot advance until s/he has mastered the Topic or Concept. That's an A.

There is No Bell Curve when SPIKE Pedagogy is practiced.

Now 80% Succeed, and only 20% Fail by quitting, and most of those simply won't do the work. Only a very few Students can't learn the material, especially in Practical Math.

Students do forget things they have learned and I recommend a lot of review. I also give periodic Review Tests to be sure they have retained their knowledge. I can only do this thanks to SPIKE Pedagogy.

OK, now on to another Adventure.

The Bell Curve got me into trouble after I earned my Ph.D. in Math. It also ultimately resulted in this Book!

Chapter 23: How the Curve Got Me Fired from the ISU Math Department

In the Spring of 1966, I was finishing my Thesis for my Ph.D. in Algebraic Number Theory at Indiana University (IU).

One morning there was a knock on the door of my apartment and my roommate was given a message for me from Dr. Vesper Moore, the Chairman of the Math Education Dept at Indiana State University. Indiana State University, ISU, (formerly Indiana State Teachers College, ISTC) is in Terre Haute, Indiana. Dr. Moore had heard about how great a Math Teacher I was and wanted to know if I would like to be a Math Professor at ISU after I got my Ph.D. DePauw did not have any openings and my Dad was dying of cancer in Greencastle.

I had already decided that I did not want to do Research Math, as the Professors in large schools like IU and Purdue were required to do. Indeed, their Careers depended on their Research, not their Teaching Ability or Performance. I was good at Teaching, but not so much so at Research.

In fact, I did not have a single Professor at IU as a graduate student that I thought was a good teacher. They were graded on their Research and the Ph.D.'s they produced.

This was in the mid 1960's and there was a dearth of Math Ph.D.s. A new Math Ph.D. was in high demand. I could have joined the faculty at many different schools. Indeed, IU expected me to join a Research Faculty at a good Research School.

ISU was NOT in that category. It was a "Teachers College". Indeed, ISU did not even have a Math Department. Their Math Dept. was a small subset of their Math Education Dept.

At IU, the Math Education Dept. was totally independent from the Math Dept., even in another building somewhere. I had never even met a Math Ed Professor or Math Ed Major at IU.

So, one bright summer day I visited ISU and Dr. Moore. The ISU Math Ed Dept. was in a beautiful new Building, much nicer than the old Swain Hall at IU. Everyone was very friendly.

I was told they needed someone to teach the advanced Math Theory courses to their few Math Majors. I could have a Six Hour Teaching Load.

In all other Research schools, a beginning Ph.D. was an Assistant Professor and then later promoted to Associate Professor once tenure was achieved. The typical starting salary for a new Assistant Professor was $9,000 to $9,500 for the year.

I was offered an Associate Professor position for $11,000 for the year with another 30% if I would teach a summer class, totaling $14,100. That is $125,000 in 2022 Dollars. WOW!

All I wanted to do was teach the advanced theory courses anyway, which most new Professors couldn't do in most Research schools. I did not want to do Research for various reasons. Teaching - Yes, Research – No.
So, I took the job.

I only taught Two Hours in the Morning, three days per week, M,W,F. The rest of the time I spent talking to the other Math Ed Professors in the common meeting room or tutoring my students during Office Hours.

I did teach one Course required of all Math Ed Majors, M412, Introduction to Modern Algebra. The Math Ed Majors most feared advanced math course.

Other than that, I could teach whatever I wanted to teach. So I taught Subjects I wanted to learn better myself like Topology and Functional Analysis, etc.

Dr. Moore introduced me all around Terre Haute. I was his Pride and Joy. He visited me almost every evening after school and became a second father figure. I spent time with him on his farm. We also went to some Math Conferences together.

I did visit my dying father almost every day. Greencastle was only an hour away from Terre Haute. I had just married my wonderful wife and she enrolled as an ISU student in Home Economics. We had an idyllic life. I loved the students and they seemed to love me.

When I started, ISU had about three Math Majors. By the end of my first year, about 20 Math Ed Majors switched to become Math Majors. In my second year, this was up to about 35 Math Majors. Obviously, some of the best Math Ed Majors switched to Math. I also helped recruit three more real Math Professors.

The Math Education Professors all had Ed.D.s or Doctor of Education. They didn't really know much math. Dr. Z and Dr. K were the two tenured Math Ed Professors. Others didn't have an Ed.D.

I would talk to them all the time about Math Education and what we were teaching our future High School teachers. Often, my students would tell me something one of them had taught that was just plain wrong. When confronted, they would usually deny it or defend it as a matter of opinion. Well, Pi is a transcendental number and that is not an opinion!

Our biggest disagreement was about the Bell Curve I discussed in the last Chapter. I tried to convince them that this was an inappropriate way to grade students. They adamantly disagreed.

My father died in the late summer of 1968 as I was about to enter my third year at ISU. Of course, I was very upset and missed him. Then in September, I went into Dr. Moore's Office and he was crying.

“Wow, Doc what's the matter,” I asked.

“I have to do something I don't want to do,” he said and hesitated. He then told me, “This third year will be your last year at ISU. We are not going to renew you further or give you tenure. This is the decision of the tenured Math Ed faculty and the administration cannot overrule them. That's the AAUP rules.”

Wow. I was in a state of Shock. I loved the ISU students and this job. I fought it for about three months and the students signed a petition saying I was the best Math Professor at ISU.

It was a Hard Lesson learned. You Can't beat the System. So I began to look for a new job. The market was still good for Ph.D.s, and I would get a good recommendation from Dr. Moore.

Rose Poly Institute of Technology was a good engineering school in Terre Haute and I had given seminars for all the various Math faculties in Terre Haute.

Rose was indeed considered the best engineering school in the Midwest by some at the time. I got a call from the Rose Math Dept. Chairman, Herb Bailey.

“Craig, I heard you are leaving ISU. Is that true?” asked Herb.

“Yep Herb, they fired me. I'm a troublemaker,” I quipped.

“Would you consider coming to Rose?” asked Herb.

“Herb, I don't know much about Engineering,” I replied.

“Craig, we need you to teach the high level Math Theory Courses for our Math and Physics majors, like you teach at ISU. Topology, Modern Algebra,Functional Analysis, etc. We all teach Engineering Math like Calculus and Differential Equations.”

So I went to Rose and onto my new Adventures.

Little did I suspect how amazing the new Adventures would turn out.

There is a Lot more to Math than just Learning and Teaching Math.

You Apply Math to all sorts of Life's Adventures!

Science and Engineering to be sure.

But also Businesses of all kinds.

Little did I even suspect what the Future would hold for me!

Looking back, I thank Dr. K and Dr. Z for firing me!

Although I still feel sorry for the ISU Students and all the High School Students their Program ultimately affected.

Chapter 24: Business Applications – Hoosier Auto Parts to Triad Math

In the spring of 1969 after I thrilled the ISU Math Ed Profs by accepting an Associate Professor position at Rose Poly, my younger brother, Mike, approached me with a question.

Mike is very smart. He never went to college. He started working in the Ford Parts Dept. right out of high school after marrying his sweetheart. That was about 1961, 8 years earlier. I spent a lot of time with Mike and his family and considered his two children sort of like mine too. Mike was now working as the Asst. Parts Manager at a big Ford Dealership, Ed Martin Ford, in Indianapolis.

He said that he thought we could both make a lot of money IF we could start or buy an Auto Parts Store somewhere. But he didn't know how to buy one and didn't have any money. He knew that I had saved some money and the Hane Family had about $20,000 considering what little our father had left our mother.

Could we buy a parts store for $20,000? Maybe with a little financing from a Bank or someone maybe $50,000 or $60,000? Math! Could I apply my Math knowledge to make it happen? Neither Mike or I had any real business experience. I knew some Math I could apply to business analysis and Mike knew how to run an auto parts store.

I began to ask around and try to find a Parts Store to buy. We looked at several but could not find a deal we could or would do. Then, a friend told me that a guy named Scotty Webber had a big parts store for sale, Hoosier Auto Parts.

I had never seen the store even though it was in Terre Haute down near the Wabash River on 2nd Street.

I called Scotty and we met for lunch and I told him about Mike, who as a Ford guy also knew a lot about Autolite, a Ford subsidiary. Turns out Scotty had a problem with Autolite and that interested him in Mike.

Scotty asked me how much we would like to invest. I hesitated and said, "Well we have $20,000 Cash and with some financing perhaps $50,000 or $60,000."

"Get that Number right out of your head," growled Scotty! He was intimidating.

"OK," I said, "What could be a deal for us?" I sensed Scotty was really eager to sell for some reason. He was in his early 50's and had started this store right after WWII about 25 years earlier and it had grown from there.

"$20,000 Down and $1,500/month for ten years and I will rent you the building for $400/month," responded Scotty immediately.

Wow. $200,000 for the store and rent of $400/month.

Scotty went on to say, "I will keep all the Accounts Receivables and Payables the day we close the deal and transfer ownership. And, NO Inventory. No looking over our past financial books. You buy it 'as is'."

WOW. I called Mike and we scheduled a visit to see the store.

I talked to a business professor I knew at ISU and he told me it was unheard of to buy a business without looking at the financial books or taking an inventory. "Something was fishy or wrong," he said.

Mike and I visited the Hoosier Auto Parts Store. Wow. Huge. Lots of Inventory. 10 or 15 employees and lots of customers coming and going while we were there.

“How valuable is the Inventory?” I asked Mike.

“Don't know. It's huge, but with a lot of obsolete parts that are unsellable,” Mike said.

I asked him, “Guess the size.”

“Oh, probably at least $100,000, could be $200,000 or more,” Mike stated.

Hmm. $20,000 plus we would be paying $135,000 over ten years, if you amortize the loan at 6%/Yr. which I could do. Wasn't too easy to do back then, but I could do the Math.

Mike estimated the Sales should be at least $600/Day with about a 50% profit margin since we were going to buy it as both a Warehouse Distributor and Jobber. Complicated deal. So that would be $300/Day or $9,000/Month in Gross Profit. Estimated Payroll was about $5,000 or $6,000. We didn't know. We had seen no financials. We should be able to pay the $1,500 to Scotty and have enough for Mike to live on. Maybe. Who knows?

Well, when you go on an Adventure you often have to take some Risk. What the Heck! Our Dad had waited until he was 50 yrs old to take the plunge and then got cancer and died.

Worse case, we lose the $20,000. No big deal. I had a very high paid Job as a Rose Professor and Mike could always get a good job like he had.

Take the Plunge. Much to gain, and little to lose. What today Nassim Taleb calls a Convex Option.

We did the deal and closed it in September 1969, the same month I started teaching at Rose. Heck. It was a hectic summer!

Long story short, turns out the Sales were about $1,000/Day. Since we did not have to buy more parts and pay for them for 45 days, our Cash in the Bank quickly grew to about $50,000 within a couple of months. First surprise. Wow. My ISU Business Professor friend was shocked.

Then since we were a new Corporation as a Warehouse Distributor, our suppliers exchanged all of our obsolete parts with new parts. So our saleable inventory grew dramatically over the next six months.

But first, within thirty days of our closing, Scotty had a second store, United Auto Parts, and insisted we buy it 30 days later for no money down and $400/month. We had no choice but to do it due to some managerial problems.

Finally, after all the old inventory was replaced in about six months we finally did our own inventory. We had about $60,000 cash in the bank and the Inventory between the two stores came in at about $600,000.

So Mike and I had a Net Worth of over $400,000. About $3,000,000 in 2022 Dollars. Wow! Talk about Rags to Riches. The next year we started a third parts store, Fort Harrison Auto Parts in a building I negotiated with a No Money down deal.

Then in 1971, we built the best 1/8 Mile Dragstrip in the USA, Action Dragway, in Terre Haute. It was Sanctioned by NHRA and a huge success for two years. But, . .

Just in time for the Great Stagflation of the 1970's. It was very successful for the first two years, then went into a tailspin. I had financed it with a large SBA loan.

Mistake! NOT a Convex Option.

Scotty and his two managers we inherited decided to take back the parts stores, and Mike and I were terrible managers and business men. Mostly, due to ignorance.

Bottom line, we managed to go broke by 1976.

Yes, we had external reasons to blame, but the bottom line is we lacked the Wisdom to manage our businesses.

I had left Rose after four years in 1973 to devote full time to the businesses.

I had started a Racing Gasoline Business with a partner from Rose, Dr. Dean Hill, called H & H Racing Gasoline. This introduced us to many racing engine builders.

Dealing with race teams, I decided to develop a microcomputer system to monitor race engines on a water brake dynamometer. I called it the Hane Dynabrane. It took me three years to learn enough electronics and computer programming to make it. It was based on the Intel 4040 four bit microprocessor which had been introduced to the world in 1973 or so.

I began the development in 1974. Obviously, I have left out a lot of details that could fill a book on their own. A fantastic learning experience, that would have been impossible without the Practical Math I knew.

Long story short. I was not a good businessman. All four businesses failed. They succeeded wonderfully at first and then failed.

But, I had a lot of wonderful times and fun building them. Never any regrets.

Failure? Just, go on to the next Adventure. Like ISU to Rose and Hoosier Auto Parts.

Failure can lead to a new Success that otherwise would not have happened.

I went totally broke in about 1977, and it took me until about 1979 to recover when I taught Electronics at ISU's School of Technology for a year.

I taught their advanced course on Operational Amplifiers, Op Amps, which are the analog microprocessors in most electronic circuits at that time. I had learned about them designing the Dynabrane. Very few folks knew much about them.

I taught a four credit hour Op Amps course at ISU's School of Technology in 1979, and then realized that one could teach a technician what he needed to know about Op Amps in about two days if you did it right. No one believed me.

So, I created a two day Workshop on Troubleshooting Op Amps and delivered it to about 12 local technicians. They loved it and were amazed.

So on to my next Adventure in 1980. I started Hane Industrial Training with my first two-day Op Amp Workshop and began to market it. I was doing something I loved. Teaching folks things that would help them improve their lives via improved job performance. Sort of like teaching math. Indeed, you can "prove" the Fundamental Theorem of Calculus with two Op Amp circuits using Capacitors.

Several folks I admired told me Hane Training would fail, and I admitted they were probably right. But, I had no other thing I wanted to do more.

In the first year, I had $35,000 in sales with $10,000 to live on. Not too bad. It was more than I had made in the last few years.

In the second year, sales were $60,000 with about $20,000 to live on. Getting better.

By the third year, I developed a second workshop on Digital Electronics, which I also had learned about with the Dynabrane. Then a third workshop on microcomputers. Again from the Dynabrane experience.

Sales were about $120,000 with $50,000 to live on, or about $150,000 in 2022 Dollars. I had more expenses since I hired a sales person and another instructor. But, that's $150,000 in 2022 Dollars to live on. More than the Professors were making, with lots of travel and fun.

Hane Training grew over the years until we did $3,000,000 in sales in the year 2000 with about $600,000 in Net Profit.

Needless to say I invested in several other adventures and also had a lot of fun traveling. Actually, I had traveled all over the USA in a motorhome while teaching the workshops. I put over 100,000 miles on it, and took the family with me during the summers.

I basically “semi-retired” and had other folks running the business from about 1995 onwards. So from age 57 until age 70, I just traveled a lot, goofed off and had a lot of fun. Up the Amazon River... Over to Europe and the UK... Greece and Turkey. Lots of travel and fun with my family.

Well then something 'wonderful' happened. In 2008, when I turned 70, I got caught up in the great Bank Collapse and over the next three years, I went Broke again. I even lost our house. I only had some worthless Assets left over.

I'll admit, it didn't seem 'wonderful' at the time. But, “Keep the Faith!”

Hane Training had gone downhill since the year 2000 and was barely surviving. Markets do change! GM, Ford, and Chryler were my big customers and they were failing too. OK, what to do?

Well, My desire had always been to create a Math Education Program that would work for almost all students. So in 2008, I took some of the "worthless" technologies I had invested in and began to see IF I could create a new Online Math Program.

At first using a GoPro Camera and CD's. It sort of worked.

Then, amazingly in 2009, Wolfram Alpha appeared. Wow. This revolutionizes STEM Math and High School Math Education.

I went to work with some technologies we had left over that were of no use to anyone else and began to develop an Online Math Program.

The Bank Fights were over in 2011 after the Indiana Supreme Court ruled the Banks could lie and cheat with impunity. I lost. Turns out that the two small banks I had worked with and trusted and that had betrayed me went broke too. Karma?

Anyway, working on the Math Programs had kept me sane and somewhat comfortable for three years. By 2011, I was convinced it could be done and created Triad Math Inc.

I also wrote a book, *Teaching Math*, which outlined the Plan for a Six Tier Program. And, the rest is history.

In about 2014, I was able to convert some "worthless" intangible assets into a financial windfall that made me financially independent. Not rich. But, enough to live modestly in a bungalow in the woods, a lifestyle I love.

Sort of like life along Deer Creek, but with amazing modern technologies and amenities like air conditioning, wonderful food, the Internet with all it's technologies like Zoom, and a Smartphone, etc.

Triad Math has been my full time Mission (and Hobby) since then and I have educated many students who tell us it has "saved their lives."

God works in mysterious ways we don't understand, but just have to have faith that if we move forward and do the "right things" all will be well. Stoicism!

My latest book, *Golden Rule Math for All God's Children*, is the first book I put on Amazon. You can get a Free PDF Copy at: https://goldenrulemath.com/ or by going to www.CraigHane.com/book

Triad Math's Six Tier Program has proven to be very successful for virtually all of our students. Most of them are Homeschooled.

I have decided that now is the time to expose the Public School Math Programs for what they are, and how they are ruining our students' opportunities and ultimately the USA. Thus, this book.

BUT, Thank God there now is a Solution to this horrible Situation.

It turns out that any Parent can Homeschool a Student in Math, even if that student is attending a Private or Public School.

Any Parent can save their Student from the Public School Math Programs.

Learn How in Part V.

Part V:

A 21st Century Solution

Chapter 25: A Sales Pitch – A 21st Century Solution

Any School, Public, Private or Homeschool, can deliver an Optimal Math Education for ALL Students in 2022 thanks to many wonderful technologies, some less than a decade old.

My wish is that this book will stimulate such a transformation.

Unfortunately, I am doubtful Public Schools will make such a radical transformation in the near future due to political and financial considerations the Math Education Leaders will not overcome soon.

For example, ALL current middle and high school Math Textbooks are obsolete.

In the 21st Century Solution, the Math Textbooks for each student will cost less than $5.

Hmm! What does this do to the Math Educator Authors and Book Publishers financially? They go the way of Kodak and Blockbuster. I can understand their resistance and why they probably will condemn and criticize this book.

Maybe some independent Private Schools will make the radical transformation to an Optimal Solution. Hopefully, they will lead the way for Public Schools to make the transformation.

Fortunately, Homeschools are beginning to do so, and are enjoying MUCH Success with an Optimal 21st Century Solution that we have created and are delivering.

Homeschools may lead the way for Private Schools.

But, what is more important for You, dear Reader, is what We can do to help Students even though they are going to a Public or Private School. You should understand what we can do from the information in this Part V.

Part V is an Overview of this 21st Century Solution. It will be based on a brief explanation of Triad Math's Six Tier Math Program.

Yes, this is a "Sales Pitch" for You to help any Student that you can to obtain an Optimal 21st Century Math Education for that Student.

IF you can find a better Solution, I will be the first to cheer for you. Please let me know if that should happen.

Meantime, Educate yourself and then other Parents and Students about what you will learn in Part V.

Onward to SPIKE!

Chapter 26: SPIKE Pedagogy – A Critical Ingredient

You may learn more about SPIKE Pedagogy in Chapter 1 of another book I have written, *How and Why Homeschool Math can be vastly superior to Public School Math*.

You may get a copy at: www.CraigHane.com It is Free.

Chapter 1 is both a 19-minute Video and/or a PDF to read. So, you get an Audio/Text choice.

This Book could have been titled, *How and Why 21st Century Math Education IS Vastly Superior to 20th Century Math Education*.

Here is a brief Overview of SPIKE Pedagogy:

Self Paced – Each Student must learn Math at the Student's own Pace. This is virtually impossible in any synchronized Classroom. But it is very easy to do with Tutorial Videos along with Notes, Exercises, Quizzes and a Forum for questions.

Proper Content – Always be sure the Student has the necessary Prerequisite Math Knowledge for the current Topic being taught. Review when necessary.

Interactive – Math is learned 'by doing', and must be very interactive with both the Coach and Student.

In Triad Math' Six Tier Program, Dr. Del is the Tutor via Tutorial Videos delivered from Amazon Web Services through a Learning Management System, along with a Student Forum to ask questions and get answers.

The Coach can be anyone who is capable of Coaching. We have training on how to do that in Chapters 7, 8 and Addendum 3 in the *How and Why . . .* book mentioned above.

In many Homeschools, the Coach is the Parent or an Older Student. The Coach does not have to know the Math.

Keeping Score – The Student's Progress needs to be recognized and celebrated. This is accomplished via the Learning Management System and the Coach.

Never compare two students since one will always be faster or ahead of the other student. Recognize each Student's Progress. This helps build Self-Esteem and Self-Confidence.

Learning Math is also "Learning to Learn" that can then apply to many other subjects too, as well as 'Learning on the Job' later in life.

So it's a Double Whammy. Learn Math and Learn to Learn.

Empathy – A Student will make a lot of Mistakes. This is necessary to learn Math and we all do it. Celebrate Mistakes as Progress and evidence of Effort. We learn from our Mistakes. Humor helps too. Laugh at your Mistakes!

As I learned over many years as both a Student, a Tutor and a Teacher, SPIKE Pedagogy is the best, and for many students, the only way to learn Math.

Unfortunately, it is very difficult to achieve SPIKE Pedagogy with a Group of Students in a Classroom on a Schedule.

It is NOT the Teachers' fault. Students have very different abilities and prerequisite math backgrounds along with personal life situations. They will work at dramatically different paces for many reasons.

Review is necessary for most Students since we all tend to forget something we learned if we don't use it a lot right away.

Thanks to modern technologies You can deliver Math via SPIKE Pedagogy to any Student very cost effectively.

An expensive Tutor might work if you can afford it and find a good one, but you can also use Dr. Del via his Tutorial Videos for a few cents per hour.

Warning! SPIKE Pedagogy is necessary, but NOT sufficient for an Optimal Math Education.

You must also have Proper Math Content beyond what is required by SPIKE Pedagogy, which as you will learn in later Chapters, is very different from what is currently being taught by our Public Schools.

So, Onward to learn all about Proper Math Content in this 21st Century.

It's truly amazing!

Indeed, for advanced STEM Math, it seems like a Modern Miracle!

Chapter 27: Proper Math Content – A Six Tier Online Math Program

Proper Math Content for post-elementary Math is a Controversial Subject. Over the last hundred years, our Math Educators have evolved what I call the Standard Math Curriculum, or SMC, consisting of Math Content. Mathematics is a Huge Subject, like Music or Literature. No one Person can ever understand or learn All of it.

So, just what Math Content should we be teaching our students? I believe that Proper Math Content should include Math Concepts, Tools and Skills that will be of great value to the Student in the Student's future Life.

DO NOT burden the Student with theoretical concepts the Student will never use or be interested in, and, DO NOT force the Student to learn old Manual Tools our ancestors had to use before the Modern Tools created with today's Technologies.

WOW! That's a lot to think about. To many Parents and Students, Math is like a big Black Hole. After all…. Math is Math!

Wrong! One Set of Math Concepts and Tools can be VERY DIFFERENT than another Set of Math Concepts and Tools. It's like comparing Rock and Roll music to Gregorian Chants. After all…. Music is Music!

The Math Concepts included in the SMC taught by Public Schools today have accumulated over the last 150 years, sort of by evolution. Whether they really teach it or not, the SMC includes a lot of premature and inappropriate Math Concepts. Some are premature Theory, and some are Obsolete Manual Tools.

Of course, to a Layperson, this can be very confusing. The Math Tools the SMC still teaches includes many Obsolete Manual Tools our ancestors had to use that are now superseded by Modern Tools.

OK, what is a good Optimal 21st Century Math Program in terms of Content? Look at the Triad Math Six Tier Program Syllabus in the Appendix of this book. Then, you can compare this to any other Math Program. Of course, If you don't know much Math this won't mean much to you.

In the next six Chapters I will give an Overview of each of the Six Tiers.

Tiers 1 and 2 provide a Foundation in Practical Math for All Students.

Tier 3 is for College Bound Students getting prepared for the SAT or ACT tests, and also learning some Consumer Math and Quantitative Reasoning and Logic.

Tiers 4, 5 and 6 are for STEM Students.
Science **T**echnology **E**ngineering **M**ath.

This is Revolutionary because of an amazing 21st Century Math Tool introduced to the world in 2009, which truly revolutionizes Math for STEM Students. I think of it as a Miracle Tool. It is so difficult to believe all the Math it can do, as well as how easy it is to learn and use!

Best of All... virtually any Student can learn this new 21st Century Math on the Student's own time. This new found knowledge will be a great supplement and help in the SMC Math Program the Student's School is teaching.

Plus, a Student can try it out for No Risk financially, as you will learn, in several different ways, depending upon the Student's unique situation.

Chapter 28: Master a Scientific Calculator

An amazing thing happened 50 years ago, in 1972. It changed Math Education profoundly.

The world's first Scientific Calculator, the HP-35, was created and delivered to a very surprised set of Math Teachers and Students, although it was designed for professional engineers and scientists.

Yes, the HP-35 cost about $2,500 in 2022 Dollars ($395 in 1972 Dollars) and had a battery life of about three hours (due to LED displays). It was also difficult to use because of Reverse Polish Notation.

But, the HP-35 revolutionized how arithmetic calculations were done by Scientists and Engineers. Faster, less error prone and sometimes impossible with any old manual tools. Advanced Students who could afford it bought one. Gone were the Manual techniques and even Log and Trig Tables and the Slide Rule. Math Education improved over the next decade.

It did, however, put a lot of Math Teachers “out of business” for teaching these old, now obsolete, Manual Tools.

Today a Student can use a MUCH BETTER Scientific Calculator like the TI-30Xa, which costs about $10 in 2022 Dollars.

Coupled with SPIKE Pedagogy, a Student can Master the TI-30Xa very Quickly, Easily and Inexpensively, maybe for No Money.

Visit Special Offers on p. 163 to see how.

This will give any Student a 'taste' of how Math can be taught with modern 21st Century technologies. Plus, there are ten lessons reviewing the Rules of Arithmetic which must be mastered before proceeding into Algebra, usually the next Math Subject.

Then a Student will be empowered to learn Practical Algebra, Practical Geometry, and Practical Trigonometry in about One Semester. Believe it or Not!

Of course, it must be with Proper Content and SPIKE Pedagogy.

I know of NO Public School that teaches this Proper Content today. I do hope that changes very soon, but I am not holding my breath. This alone will revolutionize Math Education.

That is why the next Chapter might be the most important Chapter in this book for many readers and their students.

Chapter 29: A Foundation for All Math Students

Let's suppose a Student has mastered using the TI-30Xa for performing arithmetic calculations. Much better than the old difficult to learn, error prone, time consuming, manual arithmetic algorithms.

So what? What are you going to do with this new found ability?

How about solving practical problems that come up in 1,001 technical fields like landscaping, cooking, carpentry, mechanics and on and on?

What Math is required for this?

Algebra and Geometry and Trigonometry – Practical, not Theoretical.

How long do you think it will or should take to master these Three subjects?

Well, thanks to the Scientific Calculator and SPIKE Pedagogy, the answer is:

ONLY About One Semester. About 60 Hours for most Students.

Self-Paced with a lot of Review, it can be done in about 60 Hours in One Semester!

Do you know any Public Schools teaching this? I would be delighted to learn of one. Of course, I would also be delighted to help a School do this.

Anyway, to learn more about how any Student can achieve this go to Special Offers on p. 163.

How many Lessons?

10 Algebra lessons
19 Geometry lessons
8 Trigonometry lessons

Algebra isn't of much use just by itself. But it is wonderful when applied to other fields. People just don't realize they're using it. For example, in baking, doubling a recipe is essentially using algebra. Fields such as chemistry use a lot of algebra without using geometry or trigonometry.

When you combine Practical Algebra with Practical Geometry, you can solve all sorts of practical problems in many technical fields.

Why Trigonometry? Isn't it advanced and difficult?

Well, with Algebra and Geometry you can solve a lot of practical problems and even some involving a few special angles. But, you cannot solve many problems involving angles. There's just not enough Concepts and Tools! Yet.

Trigonometry is like an extension of Geometry that now empowers you to solve problems involving angles too. And Practical Trig is easy with the Scientific Calculator to do the Trig arithmetic.

Quick example. Take a Triangle, maybe a flower bed plot or a bird feeder side, with three sides of 8.6 U, 12.4 U and 16.7 U where "U" is some unit of length.

What is the Area of this triangle, and what are the measures of its three angles?

After learning Practical Trig, you will be able to solve this in less than a minute. Easy-Peasy. In fact, you will be able to solve this in two different ways to be sure you didn't make a careless error.

This is my One Question Quiz to see if an applicant knows practical math.

Answer: 103.9 deg opposite the 16.7 U side, 30.0 deg opposite the 8.6 U side, 46.1 deg opposite the 12.4 U side and the Area = 51.8 Sq U.

How much does the program cost?

Go to Special Offers on p. 163.

A Sales Pitch?

You bet! Why?

If I were a Student going to any School, then I would want to do this on my own.

Chapter 30: College Prep and Consumer Math

I consider the standardized tests SAT and ACT to be Horrible Tests. They are timed tests and only allow some basic calculators like the TI-30Xa. Taking a timed test clearly puts some students at a disadvantage, even if they know and understand the Math being tested.

These tests are designed to get a Bell Curve, see Chapter 22. The tests have several very easy problems, a few moderately difficult problems and a few very difficult problems. Indeed, a few difficult problems that a person cannot solve unless a "tricky" process, fact or solution is known.

I have a Ph.D. in Math and when I took a Sample SAT test, there were two problems out of twenty that took me many minutes to figure out how to solve. Certainly not enough time in the allotted twenty minutes. Because of this, I would have scored fairly low on the test.

However, once I learned the tricks, if I retook a similar test, I would have scored very high. Probably still not perfect since I tend to make a lot of careless errors. The test score has very little to do with my Math abilities and knowledge.

Indeed, there are many SAT Prep services and Programs that help a student learn how to solve the tricky problems quickly. So Parents who can afford such Programs will have their Student prepared to score higher than a Student who cannot afford such a program.

Graphing Calculators are obsolete today, but there are Tools on any Smartphone that solve many Math Problems easily and the way a modern STEM Pro would.

Then, there is the psychology of the student when taking a test. That alone can cause a very poor score.

There are also a bunch of Math Topics that are of no interest or value to a student who is not interested in STEM. But, gotta learn 'em to get a Scholarship or go to a good school, unless your parents can get you in some other way.

In Tier 3, I teach these topics to help a student be better prepared for these tests. After all, it's the System. Also, I try to give them some psychological counseling regarding test taking.

There are also some topics in Tier 3 that may be of interest to some folks for other reasons.

Believe it or not… A student can score very high on either of these Standard Tests and NOT be prepared to compete with a properly trained student in any STEM subject at a good University.

You will see why in the next Chapter. Yes, Unbelievable, and Unconscionable.

It involves an amazing 21st Century Math Tool that revolutionizes STEM Math, and these tests will not allow a Student to use this Tool on a test.

It's like making a Student take a test on carpentry, but not letting them use any Power Tool. They can only use Manual Tools from the 18th century, and some of them are very tricky to use without a lot of preparation.

This is one reason I believe that Public School Math is Destroying the USA, handicapping our young students and discouraging them by testing on inappropriate topics.

It keeps them from pursuing a technical career that would be very productive for them.

Math Educators… PROVE ME WRONG!

Parents… Save Your Students from the Public School's Standard Math Curriculum!

And, IF you have a Student who just might be interested in some Science or Engineering subject, the next Chapter will be the most important Chapter in this book for You.

Very Inspiring!

Chapter 31: STEM Math - PreCalculus

STEM = **S**cience **T**echnology **E**ngineering **M**ath.

Historically STEM MATH has been a huge Barrier for any Student interested in Science or Engineering.

PreCalculus consists of Algebra, Geometry, Trigonometry, Complex Numbers, Analytical Geometry and Trigonometry. This is at a much deeper level than Practical Algebra, Geometry and Trigonometry.

The Concepts are actually not too difficult for most potential STEM students. However, to do anything with these Concepts, you have to learn to solve problems that they create with these Concepts. That's the real Challenge.

Our brilliant, genius ancestors created a Plethora of Wonderful Manual Tools to solve these problems. Incredible Manual Tools. Newton, Leibniz, Euler, Guass and many others really gave us our Modern Civilization via the Science and Technology they created.

Only ONE Problem!

Most of these Manual Tools are very difficult to learn and master. Plus, they are very difficult to actually apply. It takes a very gifted person and lot's of Hard Work. It’s a Huge Barrier for most Students.

Sadly, these Manual Tools are still being taught to our Students in our Public Schools today. No, it's more than Sad.

It's Sadistic. It hurts our students, even the successful ones, and “kills" many weaker students.

Why is this so Bad?

Because there are Powerful Modern Tools that are Much Easier to Learn, Master and Apply.

It started with the HP-35 Scientific Calculator 50 years ago, in 1972. I've told this story elsewhere. It made arithmetic calculations Much Easier.

But, this Modern Math Tool, did not help very much with a lot of the PreCalculus Subjects, and very little with Calculus and Differential Equations, which are the Indispensable Subjects for Science and Engineering.

And, the Manual Tools for Calculus and Differential Equations are much more difficult to Learn and Master than the other Manual Tools. Then . . .

WOW!

In 2009, a Miracle Math Tool was released to the world that now makes solving virtually all STEM Math Problems VERY EASY. I call it a Miracle Math Tool because I still can't believe it Exists!

Yes, I have a Ph.D. in Math, (Algebraic Number Theory) so I'm not Too DUMB, but I can't even imagine How this Tool works. You'll just have to see it in action and experience it to even begin to appreciate it.

Its name is *Wolfram Alpha*:
https://www.wolframalpha.com/

Oh Yes, It's Free.

It's a creation of the brilliant genius Stephen Wolfram and his company Wolfram Research.

In Tier 4, I teach STEM PreCalculus subjects using *Wolfram Alpha*. Just look at the Syllabus for Tier 4. (See the Appendix.)

Of course, each Lesson consists of a Tutorial Video (thus Interactive), Notes, Exercises, and Quizzes. SPIKE Pedagogy!

The Implications for a STEM student are even greater for the workhorses of STEM Math, Calculus and Differential Equations.

See the next two Chapters. They'll be mind-blowing if this is new to you.

Ask your favorite Math Educator or Teacher about *Wolfram Alpha*.

Oh Yes, would You like to see *Wolfram Alpha* in Action? Siri on the iPhone is an application of *Wolfram Alpha*.

Wolfram Alpha is a multi-million instructions Computer Program using the computer language Mathematica which Wolfram also created in 1988. It runs on some big Computer somewhere and is accessible via the Internet.

Steve Jobs bundled Mathematica with the Next Computer in 1988, and this was what Tim Berners Lee used to create the World Wide Web.

It took Wolfram almost 30 years, with an ever more powerful Mathematica, to create *Wolfram Alpha*, and Jobs used Mathematica to help create the iPhone.

Wolfram Alpha revolutionizes how STEM Math can be learned and practiced. It is not in any Math Textbook I know of being used by our Public Schools.

This alone is putting the USA at a great Handicap in STEM subjects.

To me, it is like teaching “Horse and Buggy” Math in a Tesla Automobile world.

What do You think?

Chapter 32: STEM Math - Calculus

Modern Math Models for STEM are based on what are called Functions.

PreCalculus introduces many of the common Functions used in STEM Math Models: Polynomials, Rational Functions, Trig Functions, Exponential Functions, their Inverse Functions, Composite Functions and Special Functions (Infinite Series).

You try to Graph these functions to get a Pictorial Representation of them. To really study these Functions and determine their behaviors, you need a Tool called Differential Calculus.

Then you can find Maxima, Minima, Inflection Points, Roots, Asymptotes, regions of increasing and decreasing and concavity. Not Easy, but not too hard if you stick to problems that are doable. Impossible Manually for some problems that arise in STEM subjects.

This is usually what is called Calculus 1 in most Schools and Universities. Most Students can learn and master it with reasonable effort.

THEN, you need to find the Area under the Graph of a Function.

This plagued our ancestors for thousands of years and a Tool to solve this Problem was invented, or discovered, in the 1600's led by Newton in England and Leibniz in Germany.

This Tool is named the Fundamental Theorem of Calculus, or FTC. It leads to Integral Calculus, which is usually taught as Calculus 2. There's only One Problem.

The FTC is often very difficult to apply.

In Differential Calculus, you must find what is called the Derivative of a Function, and this is fairly easy to do with various Rules. A Level 2 or 3 Difficulty on a scale of 10.

In Integral Calculus, you must find what is called the Anti-Derivative of a Function, and this is often very difficult, or impossible to do. Thus, Integral Calculus is probably a Level 8 on a difficulty scale of 10

Integral Calculus is a Huge Barrier for most future STEM Students that must somehow be transcended. Many potential STEM Students fail and/or just give up.

Guess what?

Wolfram Alpha solves both Differential and Integral Calculus Problems automatically and immediately. With this Tool, Calculus is now a difficulty of about 1 or 2 on the scale of 10.

You can learn both Differential and Integral Calculus in about one semester and Much Better since you can do many more problems in a given amount of time. It's really doing these problems, and yields a deep understanding of the Concepts of Calculus.

Today the Calculus Barrier has a Huge Tunnel through it. *Wolfram Alpha*.

This is what I teach in Tier 5.

To not teach this 21st Century Math Tool, or an equivalent one if there is one, is in my judgment Criminal. It is Criminal to deprive potential STEM students of an educational path through a Now Obsolete Math Barrier.

Just go to https://stemmathmadeeasy.com and take the STEM Math Challenge.

Have any Public School Graduate with an A in Calculus go take this STEM Math Challenge. For that matter, ask any Calculus Teacher to take the Challenge.

It's so sad Math Educators… but ALL of your current Calculus Books are Obsolete.

Sorry Kodak and Polaroid… but Cameras that use Film are Obsolete.

This is a major reason Public School Math is Destroying the USA!

We must have great Scientists and Engineers and we can't keep importing them from countries that are teaching a better version of STEM Math.

Of course, I can't use any current Calculus Textbook because they all teach the old Manual Tools and not *Wolfram Alpha*.

So, in Tier 5, it's Tutorial Videos, Notes, Exercises, Quizzes and a Forum using *Wolfram Alpha* from the Get-Go!

Both Differential and Integral Calculus usually take one of my STEM Math students about One Semester to complete.

Now we can teach another important STEM Math Subject that is not taught today in a Public School because it is even more difficult than Integral Calculus, which itself is too difficult for most Schools.

Chapter 33: STEM Math – Differential Equations

The Workhorses of most STEM subjects are called Differential Equations.

A Differential Equation, or Diff Eq, is a Math Model of some Physical Process.

The Solution of a Diff Eq is a Function. Then you use Calculus to analyze the behavior of the Function. This is how you understand the Physical Process by analyzing its Math Model.

Now, the Solution of a Diff Eq might be a familiar function, but often it is what is called a Special Function, which is an Infinite Series Function.

Solving a Diff Eq is often even more difficult than solving an Integral Problem. Probably a difficulty of 9 on the 10 scale. Indeed, applying the Fundamental Theorem of Calculus involves solving the simplest kind of Diff Eq to obtain the Anti-Derivative Function of a Function!

OK. Guess what?

Wolfram Alpha solves any Diff Eq automatically and quickly. Easy – Peasy!

And, if there is not a solution with any known Function or Special Function, *Wolfram Alpha* will give you a Numerical Graphical Solution.

Oh yes, if you use *Wolfram Alpha Pro*, or *Wolfram Alpha* on the SupraComputer, then it will also show you a Step-by-Step Manual Solution, if one exists and for some reason you want to learn it.

You can learn about the SupraComputer at:
https://supracomputer.org/

or go to: www.CraigHane.com/book to get there too.

What makes it "Supra" is that it comes bundled with *Wolfram Alpha*, and an even more modern 21st Century Computer Language called Wolfram Language, which is an easy to use version of Mathematica.

Diff Eqs are what I cover in Tier 6.

A Student who works reasonably hard and is reasonably intelligent can Learn all Six Tiers in two to four years, depending on their background.

IF I were going to send a Student of mine to a good STEM University, I would be sure the Student had mastered all Six Tiers first.

Until Public Schools do this, I will tell it like it is.

They are Destroying the USA.

Part VI:
The Future of Math Education in the USA

Chapter 34: BAD NEWS!

Bad News for the Math Education Community.

Billions of Dollars of existing 'products' are Obsolete.

All current High School Math Textbooks are Obsolete.

Conventional Classroom Pedagogy is Obsolete.

Standard Math Tests are Obsolete.

Current Math Teachers are Obsolete, BUT can be converted into great Coaches and Mentors.

Math Educators will rebel and prolong any reformation of Public School Math Education, most likely, since their Books, Content and Pedagogy are obsolete.

Current 20th Century Math Programs are like “Horse and Buggy” in terms of both Pedagogy and, worse yet, Content.

The Real BAD NEWS is for our Students.

Then it is BAD NEWS for their Families.

Ultimately, this is BAD NEWS for the US Economy and then Society.

How much better off will the USA be when we are properly educating millions of our Students in the future Technologies that our future Jobs and Careers demand?

I fear this BAD NEWS will persist far too long, unless we ALL do something about it.

Hopefully, this Book will help accelerate the Transformation and adoption of an Optimal 21st Century Math Education for all of our Students.

An Optimal 21st Century Math Education Program can empower the USA to become a very powerful, benevolent country, and help the entire Sapien Species become much better off.

BTW, I will work with ANY Public or Private School to integrate my Six Tier Math Program into their Curriculum and have their Math Teachers become great Coaches and Mentors. They, too, will be learning a lot of new Math.

If I were a current High School Math Teacher, knowing what I know, I would welcome this opportunity and rejoice for both my students and myself!

On to some GOOD NEWS!

Chapter 35: GOOD NEWS!

Our Public Schools could deliver good 21st Century Math Education. 20th Century Math Programs are like "Horse and Buggies" Technologies. Modern 21st Century Math Programs are like "Automobiles" Technologies.

The Costs of Delivering a 21st Century Math Education like Triad Math's Six Tier Program is probably about 20% as much as the current Public School Math Programs.

Just imagine what an 80% Savings could do for our Educational System and our Tax System!

When will Public Schools modernize their Math Programs into the 21st Century? I don't know. Hopefully, Private Schools will lead the way sooner rather than later.

BUT, right Now, TODAY, You can start using Triad Math's Six Tier Program for any Students you control. Even if they are enrolled in another Math Program.

In the long term, this will probably save the student time. They will certainly learn More and Better Math. And, it is very affordable.

Conclusions – Action Plan

Go to Special Offers on p. 163.

Join me in helping give Any Student an Optimal 21st Century Math Education.

Thank You.

Appendix – The Six-Tier Syllabus

Tier 1

Tier 1 TI-30XA Calculator

TI-30XA Introduction

C1 ON/OFF FIX DEG M1 M2 M3
C2 Real Numbers: Add + Subtract – Equal =
C3 Negative Numbers + ≈ -
C4 Multiply × Divide ÷
C5 Percentage %
C6 Memory M1, M2, M3 STO RCL ( )
C7 X^2 Square
C8 √X Square Root
C9 1/X Reciprocal "Flip It"
C10 Fractions $A^{B/C}$ + - × ÷
C11 D/C Proper/Improper Fractions
C12 F ↔ D Fraction to Decimal Conversion
C13 DEG RAD GRAD Three Angle Measures
C14 SIN SIN^{-1}
C15 COS COS^{-1}
C16 TAN TAN^{-1}

Tier 1 Pre-Algebra

Pre-Algebra Introduction

P1 Real Numbers, Integers, and Rationals
P2 The Number Line, Negative Numbers
P3 Rules of Addition + -
P4 Rules of Multiplication × ÷
P5 Distributive Law + and × Combined
P6 Fractions, A/B and C/D, Rules
P7 Squares X^2 X Squared
P8 Square Roots √X
P9 Reciprocal 1/X $X \neq 0$
P10 Exponents Y^X $Y > 0$, X Can Be Any Number

Tier 2

Tier 2 Algebra

Introduction to Algebra

A1 Four Ways to Solve an Algebra Equation
A2 The Rule of Algebra
A3 X + A = B This is an Easy Linear Equation
A4 AX = B This is an Easy Linear Equation
A5 AX + B = CX + D This is an Easy Linear Equation
A6 A/X = C/D This is an Easy Linear Equation
A7 AX2 = B This is an Easy Non-Linear Equation
A8 A√X = B This is an Easy Non-Linear Equation
A9 (1) SIN X° = A, -1 ≤ A ≤ 1, or (2) $SIN^{-1}X = A°$, 0 ≤ A° ≤ 180°
A10 (1) COS X° = A, -1 ≤ A ≤ 1, or (2) $COS^{-1}X = A°$, 0 ≤ A° ≤ 180°

Tier 2 Geometry

Introduction to Geometry

G1 What is Geometry?
G2 Straight Lines and Angles
G3 Parallel Lines
G4 Triangles, Definition, Sum of Angles
G5 Pythagorean Theorem
G6 Similar Triangles
G7 Quadrilaterals, Polygons, Perimeters (P)
G8 Area of Triangles and Rectangles
G9 Formulas for Polygons
G10 Circles π Circumference
G11 Circles Area A=πr2
G12 Circles Special Properties
G13 Surface Area Blocks and Cylinders
G14 Surface Area Cones
G15 Volume Blocks and Cylinders
G16 Volume Cones
G17 Surface Area Ball or Sphere
G18 Volume Ball or Sphere, Archimedes Tombstone
G19 When Geometry is not Enough for Triangles

Tier 2 Trigonometry

Introduction to Trigonometry

T1 Trig Functions SIN COS TAN

T2 SIN X, Sine of X, X is an Angle (Degrees °)

T3 COS X, Cosine of X, X is an Angle (Degrees °)

T4 TAN X, Tangent of X, X is an Angle (Degrees °)

T5 Warning about SIN^{-1}

T6 Law of Sines

T7 Law of Cosines – Generalized Pythagorean Theorem

T8 Trigonometry Beyond Practical Math (Optional)

Tier 3

Part 1 of Tier 3 should prepare you for a standard test you will need to pass to graduate from high school.

Tier 3 Part 1

T3 Part 1 Introduction
T3 P1 L1 The Real Number System (Simmons pp. 34-36)
T3 P1 L2A Notation and Rules (Simmons pp. 36 -39)
T3 P1 L2B Notation and Rules (Simmons pp. 36 –39)
T3 P1 L3 Integral Exponents (Simmons pp. 39 –40)
T3 P1 L4 Root, Radical, Fractional Exponents (Simmons pp. 40 –43)
T3 P1 L5 Polynomials (Simmons pp. 43-45)
T3 P1 L6 Factoring Polynomials (Simmons pp. 45 –46)
T3 P1 L7 Linear Equations & Rule of Algebra (Simmons pp. 46 –49)
Plus: Review of Algebra and Rules from the Tier 2 Practical Math Foundation
T3 P1 L8 Quadratic Equation (Simmons pp. 46 –49)
T3 P1 L9 Inequalities and Absolute Values (Simmons pp. 49 –50)
T3 P1 L10 Coordinates in a Plane (Simmons pp. 53 –54)
T3 P1 L11 Functions and Graphs (Simmons pp. 51 –53)
T3 P1 L12 Straight Lines & Linear Functions (Simmons pp. 55 –56)
T3 P1 L13 Parallel and Perpendicular Lines (Simmons pp. 55 –56)
T3 P1 L14 Intersecting Straight Lines (Custom Training) You will learn a process you should master by practice.

Part 2 of Tier 3 will teach you additional mathematics you will need to excel on the SAT and ACT and other exams.

Tier 3 Part 2

T3 P2 L1 Prime Numbers (Custom Notes)
T3 P2 L2 Number Facts and Ideas (Custom Notes)
T3 P2 L3 Percents and Percentage (Custom Notes)
T3 P2 L4 Chain Discounts (Custom Notes)
T3 P2 L5 Markups and Discounts (Custom Notes)
T3 P2 L6 Means, Medians, Averages (Custom Notes)
T3 P2 L7 Ratios and Proportions (Custom Notes)
T3 P2 L8 Logic (Custom Notes)
T3 P2 L9 Arithmetic Progressions (Simmons pp. 77)
T3 P2 L10 Geometric Progressions (Simmons pp. 74 –76)
T3 P2 L11 Geometric Series (Simmons pp. 74 –76)
T3 P2 L12 Permutations and Combinations (Simmons pp. 78 –81)
T3 P2 L13 Combinations (continued) (Simmons pp. 78 – 81)
T3 P2 L14 Probability (Custom Notes)

Tier 3 Part 3: SAT/ACT Preparation

T3 P2 L1 Pep Talk
T3 P2 L2 Test Preparation
T3 P2 L3 Test Techniques
T3 P2 L4 Sample Problems A
T3 P2 L5 Sample Problems B
T3 P2 L6 Sample Problems C
T3 P2 L7 Sample Problems D
T3 P2 L8 More Fun
T3 P2 L9 Fun & Games

Tier 4

Precalculus Mathematics in a Nutshell, and Notes will be used. Geometry, Algebra, Trigonometry, and Complex Numbers, with Wolfram-Alpha will be covered.

T4I Introduction to Tier 4, and Overview

Tier 4 Geometry

G1 Introduction to Geometry Overview (pp. 2-3)
G2 Triangles: Angles, Parallel Lines, Area (pp. 4-5)
G3 Triangles: Similar Congruent (p. 6)
G4 Pythagorean Theorem (pp. 6-7)
G5 Circles: Pi, Area, Sector (pp. 7-8)
G6 Circles: Inscribed angles (pp. 8-9)
G7 Circles: Tangents & Constructions (Notes)
G8 Angles: Bisect, Trisect, Compass, Impossibilities (Notes)
G9 Cylinder: Area, Volume (pp. 9-10)
G10 Cone: Overview (pp. 10-11)
G11 Cone: Problems – Help (pp. 21-22)
G12 Cone: Optional Proof for Math Majors (Simmons)
G13 Sphere: Volume and Area, Problems (pp. 22–23)
G14 Sphere: Optional Proof with Cavalieri's Principle (pp. 13-14)

Interlude #1

Tier 4 Algebra

A1 Introduction to Algebra, Rules of Algebra Review (p. 33)
A2 Basics: Numbers (pp. 34-35)
A3 Review – Overview Tier 3 (pp. 36-50)
A4 Review – Overview Tier 3 (pp. 51-56)
*A5 Introduction to Wolfram-Alpha (Notes)
A6 Circles (pp. 57-58)
A7 Ellipses (Notes)
A8 Parabolas (pp. 58-60)
A9 Hyperbolas (Notes)
A10 Conic Sections

A11 Functions and Graphs (pp. 60-62)
A12 Polynomial Division (pp. 65-67)
A13 Logarithms Calculator (pp. 63-65)
A14 Logarithms Exponents (Notes)
A15 Examples Log Scale

Interlude #2

Tier 4 Trigonometry
T1 Introduction to Trigonometry (pp. 92-93)
T2 Review of some Analytical Geometry (pp. 93-96)
T3 Radian Measure (pp. 96-98)
T4 Trig Functions Circle Definition (pp. 98-100)
T5 Trig Identities Intro (pp. 100-101)
T6 Evaluating Trig Functions (pp. 101-103)
T7 Trig functions graphs (pp. 103-105)
T8 Frequency and Phase (Notes)
T9 Identities pp 105-6 sec 4 (pp. 114-5)
T10 Identities and Graphs (Notes)
T11 Proofs of Identities – Appendix B (pp. 111-112)
T12 Inverse Trig Functions (pp. 107-109)
T13 Law of Sines and Cosines (p. 109)

Tier 4 Complex Numbers
Complex Numbers will be treated with a modern geometric approach. Real Numbers correspond to points on a straight line Complex Numbers correspond to points in the plane. Complex Numbers have many wonderful geometric properties that relate geometry and algebra. Trigonometry is more fully understood when one understands complex numbers. Euler's identity is the key to this. Complex numbers are very powerful and indispensable in modern STEM subjects.

C1 Real Numbers Synopsis
C2 Complex Number Definition
C3 Complex Numbers Geometry
C4 Complex Number Geometry Proof
C5 Interlude for Inspiration y^x

C6 Interlude Preparation
C7 Wonderful Equation
C8 Motivation for Wonderful Equation
C9 Roots of Unity
C10 Clocks and Frequency
C11 Exponents and Logarithms

Tier 4 Algebra Special Topics
AST1 Mathematical Induction (pp. 83-84)
AST2 Progressions, Permutations, Combinations Review Tier3 (pp. 74-80)
AST3 Binomial Theorem (pp. 81-82)
AST4 Linear Equations Determinants (pp. 68-70)
AST5 Linear Equations 3D (pp. 71-73)
AST6 Cone and Sphere, Calculus Preview (pp. 84-87)

Tier 4 Geometry Special Topics for Math Majors/Teachers
GST1 Review of Geometry
GST2 Ceva's Theorem (pp. 27-29)
GST3 Heron's & Brahmagupta's Formulae (p. 18, Problem 20, pp. 30-31)
GST4 Geometry and Algebra, Analytical Geometry
GST5 Euclid Geometry vs Non-Euclidean Geometries
GST6 Calculus Preview

Tier 5

Tier 5 Part 1 Differential Calculus

T5 C1 Introduction to Calculus
- Approach to Learning Calculus
- Calculus Overview

T5 C2 Functions
- Graph Terms for Functions
- Function Graph Terms Sheet for Calculus
- Function Graphs #1 Worksheet
- Examples of Graphs

T5 C2a Functions
- More Examples

T5 C3 Derivative
- Differential Calculus
- Infinitesimals
- Derivative definitions
- Examples

T5 C4 Derivative Examples
- From definition
- From Wolfram Alpha

T5 C5 Applications to Graphing
- Increasing/Decreasing
- Max/Min
- Points of Inflection
- Concavity

T5 C6 Derivative Rules
- Linear combination Rule
- Leibniz Rule
- Quotient Rule

T5 C7 Chain Rule
- Derivative Examples from Rules
- Wolfram Alpha examples

T5 C8 Implicit Differentiation

T5 C9 Relative Rates of Change

T5 C10a Inverse Functions Basics

T5 C10 Inverse Functions

T5 C11 Series Expansions

T5 C12 Final Thoughts on Derivatives

Tier 5 Part 2 Integral Calculus

T5 C13 Integral Calculus Overview
T5 C14 Definition of Integral and the FTC
T5 C15 Techniques of Integration Overview
T5 C16 Applications of Integration Areas
T5 C17 Applications of Integration Arc Length
T5 C18 Applications of Integration Volumes Disc
T5 C19 Applications of Integration Volumes Shell
T5 C20 Applications of Integration Surface Areas
T5 C21 Parametric Functions Graphs
T5 C22 Parametric Functions Arc Length
T5 C23 Parametric Functions Tangent Line
T5 C24 Parametric Functions Area
T5 C25 Wolfram Alpha Commands
T5 C26 Improper Integrals Vertical Asymptotes
T5 C27 Improper Integrals Horizontal Asymptotes
T5 C28 Improper Integrals H A continued
T5 C29 Surface areas of solids revisited
T5 C30 Wolfram Alpha Modern STEM Tool

Tier 6

Tier 6 Part 1 Differential Equations

T6 DE0 Instructions and Advice
T6 DE1 Introduction to Differential Equations
T6 DE2 First Order Differential Equations
T6 DE3 First Order Linear Differential Equations
T6 DE4 First Order Linear Differential Equations II
T6 DE5 Polynomial Approximations of Functions
T6 DE6 Math Models (MMs) – Chimera or Reality?
T6 DE7 First Order Differential Equations – Separation of Variables
T6 DE8 First Order Linear Differential Equations Redux
T6 DE9 Second Order Linear Differential Equations
T6 DE10 Second Order Linear Differential Equations – Constant Coefficients
T6 DE11 First Order Non-Linear ODEs
T6 DE12 Differential Equation Tricks
T6 DE13 Orthogonal Curves
T6 DE14 Polar Coordinates and ODEs

Free Resources

Simply go to www.CraigHane.com to get the current free resources available from Dr. Del.

The Video Library Tab will yield many videos Dr. Del has created, and is an ever expanding Library.

These videos are all YouTube videos.

Potential STEM students will want to watch the three videos on the Concepts of Calculus.

Some are also YouTube videos of others that Dr. Del finds inspirational and informative.

Special Offers

Go to www.TriadMathInc.com/SO for the current Special Offers.

Dr. Del and Triad Math, Inc. like to give students and families actual training so they can evaluate the methods our Programs use to determine if they would then benefit from some of our training products.

Seeing is believing.

Your experience is the only one that counts for you.

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