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Tiers 1-6: Syllabus

Tier 1

Tier 1 TI-30XA Calculator

TI-30XA Introduction

- C1 ON/OFF FIX DEG M1 M2 M3
- C2 Real Numbers: Add + Subtract - Equal =
- C3 Negative Numbers + \approx -
- C4 Multiply \times Divide \div
- C5 Percentage %
- C6 Memory M1, M2, M3 STO RCL ()
- C7 X^2 Square
- C8 \sqrt{X} Square Root
- C9 $1/X$ Reciprocal "Flip It"
- C10 Fractions $A^{B/C}$ + - \times \div
- C11 D/C Proper/Improper Fractions
- C12 F \leftrightarrow D Fraction to Decimal Conversion
- C13 DEG RAD GRAD Three Angle Measures
- C14 SIN SIN^{-1}
- C15 COS COS^{-1}
- C16 TAN TAN^{-1}

Tier 1 Pre-Algebra

Pre-Algebra Introduction

- P1 Real Numbers, Integers, and Rationals
- P2 The Number Line, Negative Numbers
- P3 Rules of Addition + -

- P4 Rules of Multiplication $\times \div$
- P5 Distributive Law $+$ and \times Combined
- P6 Fractions, A/B and C/D , Rules
- P7 Squares X^2 X Squared
- P8 Square Roots \sqrt{X}
- P9 Reciprocal $1/X$ $X \neq 0$
- P10 Exponents Y^X $Y > 0$, X Can Be Any Number

Tier 2

Tier 2 Algebra

Introduction to Algebra

- A1 Four Ways to Solve an Algebra Equation
- A2 The Rule of Algebra
- A3 $X + A = B$ This is an Easy Linear Equation
- A4 $AX = B$ This is an Easy Linear Equation
- A5 $AX + B = CX + D$ This is an Easy Linear Equation
- A6 $A/X = C/D$ This is an Easy Linear Equation
- A7 $AX^2 = B$ This is an Easy Non-Linear Equation
- A8 $A\sqrt{X} = B$ This is an Easy Non-Linear Equation
- A9 (1) $\text{SIN } X^\circ = A$, $-1 \leq A \leq 1$, or (2) $\text{SIN}^{-1}X = A^\circ$, $0 \leq A^\circ \leq 180^\circ$
- A10 (1) $\text{COS } X^\circ = A$, $-1 \leq A \leq 1$, or (2) $\text{COS}^{-1}X = A^\circ$, $0 \leq A^\circ \leq 180^\circ$

Tier 2 Geometry

Introduction to Geometry

- G1 What is Geometry?
- G2 Straight Lines and Angles
- G3 Parallel Lines
- G4 Triangles, Definition, Sum of Angles
- G5 Pythagorean Theorem
- G6 Similar Triangles
- G7 Quadrilaterals, Polygons, Perimeters (P)
- G8 Area of Triangles and Rectangles
- G9 Formulas for Polygons
- G10 Circles π Circumference
- G11 Circles Area $A = \pi r^2$
- G12 Circles Special Properties
- G13 Surface Area Blocks and Cylinders
- G14 Surface Area Cones
- G15 Volume Blocks and Cylinders
- G16 Volume Cones
- G17 Surface Area Ball or Sphere

G18 Volume Ball or Sphere, Archimedes Tombstone

G19 When Geometry is not Enough for Triangles

Tier 2 Trigonometry

Introduction to Trigonometry

T1 Trig Functions SIN COS TAN

T2 SIN X, Sine of X, X is an Angle (Degrees °)

T3 COS X, Cosine of X, X is an Angle (Degrees °)

T4 TAN X, Tangent of X, X is an Angle (Degrees °)

T5 Warning about SIN^{-1}

T6 Law of Sines

T7 Law of Cosines – Generalized Pythagorean Theorem

T8 Trigonometry Beyond Practical Math (Optional)

Tier 3

Tier 3 Part 1

T3 Part 1 Introduction

T3 P1 L1 The Real Number System (Simmons pp. 34-36)

T3 P1 L2A Notation and Rules (Simmons pp. 36 -39)

T3 P1 L2B Notation and Rules (Simmons pp. 36 -39)

T3 P1 L3 Integral Exponents (Simmons pp. 39 -40)

T3 P1 L4 Root, Radical, Fractional Exponents (Simmons pp. 40 -43)

T3 P1 L5 Polynomials (Simmons pp. 43-45)

T3 P1 L6 Factoring Polynomials (Simmons pp. 45 -46)

T3 P1 L7 Linear Equations & Rule of Algebra (Simmons pp. 46 -49)

Plus: Review of Algebra and Rules from the Tier 2 Practical Math Foundation

T3 P1 L8 Quadratic Equation (Simmons pp. 46 -49)

T3 P1 L9 Inequalities and Absolute Values (Simmons pp. 49 -50)

T3 P1 L10 Coordinates in a Plane (Simmons pp. 53 -54)

T3 P1 L11 Functions and Graphs (Simmons pp. 51 -53)

T3 P1 L12 Straight Lines & Linear Functions (Simmons pp. 55 -56)

T3 P1 L13 Parallel and Perpendicular Lines (Simmons pp. 55 -56)

T3 P1 L14 Intersecting Straight Lines (Custom Training)

You will learn a process you should master by practice.

Part 1 of Tier 3 should prepare you for a standard test you will need to pass to graduate from high school.

Part 2 of Tier 3 will teach you additional mathematics you will need to excel on the SAT and ACT and other exams.

Tier 3 Part 2

- T3 P2 L1 Prime Numbers (Custom Notes)
- T3 P2 L2 Number Facts and Ideas (Custom Notes)
- T3 P2 L3 Percents and Percentage (Custom Notes)
- T3 P2 L4 Chain Discounts (Custom Notes)
- T3 P2 L5 Markups and Discounts (Custom Notes)
- T3 P2 L6 Means, Medians, Averages (Custom Notes)
- T3 P2 L7 Ratios and Proportions (Custom Notes)
- T3 P2 L8 Logic (Custom Notes)
- T3 P2 L9 Arithmetic Progressions (Simmons pp. 77)
- T3 P2 L10 Geometric Progressions (Simmons pp. 74 –76)
- T3 P2 L11 Geometric Series (Simmons pp. 74 –76)
- T3 P2 L12 Permutations and Combinations (Simmons pp. 78 –81)
- T3 P2 L13 Combinations (continued) (Simmons pp. 78 –81)
- T3 P2 L14 Probability (Custom Notes)

Tier 3 Part 3: SAT/ACT Preparation

- T3 P2 L1 Pep Talk
- T3 P2 L2 Test Preparation
- T3 P2 L3 Test Techniques
- T3 P2 L4 Sample Problems A
- T3 P2 L5 Sample Problems B
- T3 P2 L6 Sample Problems C
- T3 P2 L7 Sample Problems D
- T3 P2 L8 More Fun
- T3 P2 L9 Fun & Games

Tier 4

Precalculus Mathematics in a Nutshell, and Notes will be used. Geometry, Algebra, Trigonometry, and Complex Numbers, with Wolfram-Alpha will be covered.

T4I Introduction to Tier 4, and Overview

Tier 4 Geometry

- G1 Introduction to Geometry Overview (pp. 2-3)
- G2 Triangles: Angles, Parallel Lines, Area (pp. 4-5)
- G3 Triangles: Similar Congruent (p. 6)
- G4 Pythagorean Theorem (pp. 6-7)
- G5 Circles: Pi, Area, Sector (pp. 7-8)
- G6 Circles: Inscribed angles (pp. 8-9)

- G7 Circles: Tangents & Constructions (Notes)
- G8 Angles: Bisect, Trisect, Compass, Impossibilities (Notes)
- G9 Cylinder: Area, Volume (pp. 9-10)
- G10 Cone: Overview (pp. 10-11)
- G11 Cone: Problems – Help (pp. 21-22)
- G12 Cone: Optional Proof for Math Majors (Simmons)
- G13 Sphere: Volume and Area, Problems (pp. 22–23)
- G14 Sphere: Optional Proof with Cavalieri’s Principle (pp. 13-14)

Interlude #1

Tier 4 Algebra

- A1 Introduction to Algebra, Rules of Algebra Review (p. 33)
- A2 Basics: Numbers (pp. 34-35)
- A3 Review – Overview Tier 3 (pp. 36-50)
- A4 Review – Overview Tier 3 (pp. 51-56)
- *A5 Introduction to Wolfram-Alpha (Notes)
- A6 Circles (pp. 57-58)
- A7 Ellipses (Notes)
- A8 Parabolas (pp. 58-60)
- A9 Hyperbolas (Notes)
- A10 Conic Sections
- A11 Functions and Graphs (pp. 60-62)
- A12 Polynomial Division (pp. 65-67)
- A13 Logarithms Calculator (pp. 63-65)
- A14 Logarithms Exponents (Notes)
- A15 Examples Log Scale

Interlude #2

Tier 4 Trigonometry

- T1 Introduction to Trigonometry (pp. 92-93)
- T2 Review of some Analytical Geometry (pp. 93-96)
- T3 Radian Measure (pp. 96-98)
- T4 Trig Functions Circle Definition (pp. 98-100)
- T5 Trig Identities Intro (pp. 100-101)
- T6 Evaluating Trig Functions (pp. 101-103)
- T7 Trig functions graphs (pp. 103-105)
- T8 Frequency and Phase (Notes)
- T9 Identities pp 105-6 sec 4 (pp. 114-5)
- T10 Identities and Graphs (Notes)
- T11 Proofs of Identities – Appendix B (pp. 111-112)

T12 Inverse Trig Functions (pp. 107-109)

T13 Law of Sines and Cosines (p. 109)

Tier 4 Complex Numbers

Complex Numbers will be treated with a modern geometric approach. Real Numbers correspond to points on a straight line Complex Numbers correspond to points in the plane. Complex Numbers have many wonderful geometric properties that relate geometry and algebra. Trigonometry is more fully understood when one understands complex numbers. Euler's identity is the key to this. Complex numbers are very powerful and indispensable in modern STEM subjects.

C1 Real Numbers Synopsis

C2 Complex Number Definition

C3 Complex Numbers Geometry

C4 Complex Number Geometry Proof

C5 Interlude for Inspiration y^x

C6 Interlude Preparation

C7 Wonderful Equation

C8 Motivation for Wonderful Equation

C9 Roots of Unity

C10 Clocks and Frequency

C11 Exponents and Logarithms

Tier 4 Algebra Special Topics

AST1 Mathematical Induction (pp. 83-84)

AST2 Progressions, Permutations, Combinations Review Tier3 (pp. 74-80)

AST3 Binomial Theorem (pp. 81-82)

AST4 Linear Equations Determinants (pp. 68-70)

AST5 Linear Equations 3D (pp. 71-73)

AST6 Cone and Sphere, Calculus Preview (pp. 84-87)

Tier 4 Geometry Special Topics for Math Majors/Teachers

GST1 Review of Geometry

GST2 Ceva's Theorem (pp. 27-29)

GST3 Heron's & Brahmagupta's Formulae (p. 18, Problem 20, pp. 30-31)

GST4 Geometry and Algebra, Analytical Geometry

GST5 Euclid Geometry vs Non-Euclidean Geometries

GST6 Calculus Preview

Tier 5

Tier 5 Part 1 Differential Calculus

- T5 C1 Introduction to Calculus
 - Approach to Learning Calculus
 - Calculus Overview
- T5 C2 Functions
 - Graph Terms for Functions
 - Function Graph Terms Sheet for Calculus
 - Function Graphs #1 Worksheet
 - Examples of Graphs
- T5 C2a Functions
 - More Examples
- T5 C3 Derivative
 - Differential Calculus
 - Infinitesimals
 - Derivative definitions
 - Examples
- T5 C4 Derivative Examples
 - From definition
 - From Wolfram Alpha
- T5 C5 Applications to Graphing
 - Increasing/Decreasing
 - Max/Min
 - Points of Inflection
 - Concavity
- T5 C6 Derivative Rules
 - Linear combination Rule
 - Leibniz Rule
 - Quotient Rule
- T5 C7 Chain Rule
 - Derivative Examples from Rules
 - Wolfram Alpha examples
- T5 C8 Implicit Differentiation
- T5 C9 Relative Rates of Change
- T5 C10a Inverse Functions Basics
- T5 C10 Inverse Functions
- T5 C11 Series Expansions
- T5 C12 Final Thoughts on Derivatives

Tier 5 Part 2 Integral Calculus

- T5 C13 Integral Calculus Overview
- T5 C14 Definition of Integral and the FTC
- T5 C15 Techniques of Integration Overview
- T5 C16 Applications of Integration Areas
- T5 C17 Applications of Integration Arc Length
- T5 C18 Applications of Integration Volumes Disc
- T5 C19 Applications of Integration Volumes Shell
- T5 C20 Applications of Integration Surface Areas
- T5 C21 Parametric Functions Graphs
- T5 C22 Parametric Functions Arc Length
- T5 C23 Parametric Functions Tangent Line
- T5 C24 Parametric Functions Area
- T5 C25 Wolfram Alpha Commands
- T5 C26 Improper Integrals Vertical Asymptotes
- T5 C27 Improper Integrals Horizontal Asymptotes
- T5 C28 Improper Integrals H A continued
- T5 C29 Surface areas of solids revisited
- T5 C30 Wolfram Alpha Modern STEM Tool

Tier 6

Tier 6 Part 1 Differential Equations

- T6 DE0 Instructions and Advice
- T6 DE1 Introduction to Differential Equations
- T6 DE2 First Order Differential Equations
- T6 DE3 First Order Linear Differential Equations
- T6 DE4 First Order Linear Differential Equations II
- T6 DE5 Polynomial Approximations of Functions
- T6 DE6 Math Models (MMs) – Chimera or Reality?
- T6 DE7 First Order Differential Equations – Separation of Variables
- T6 DE8 First Order Linear Differential Equations Redux
- T6 DE9 Second Order Linear Differential Equations
- T6 DE10 Second Order Linear Differential Equations – Constant Coefficients
- T6 DE11 First Order Non-Linear ODEs
- T6 DE12 Differential Equation Tricks
- T6 DE13 Orthogonal Curves
- T6 DE14 Polar Coordinates and ODEs