Galileo bats 50% in this story.

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Trajectory of a Ball

Galileo thought that a ball would follow a parabolic trajectory when thrown. He was correct.

This contradicted Aristotle who thought the trajectory was a circle.

To compare them, consider the parabola $y = 1 - x^2$ and the circle $y = (1 - x^2)^{1/2}$

WA 1 Plot 1-x^2,(1-x^2)^.5 from x = -1 to 1

WA 2 Arc Length 1-x^2, from x = -1 to 1

Ans. 2.9579

WA 3 Arc Length (1-x^2)^.5 , from x = -1 to 1

Ans. 3.1416

But, Aristotle would have seen this discrepancy and also realized that the tangent to the circle was vertical, which it would be if the ball went in an arc.

So, he might have used a better example. Can you see a possible one?

Maybe we should throw the ball so its peak is less than half the distance we are throwing it. Say pass through the points (-1,0), (0,.5),(1,0)

Then the parabola would be $y = 1/2 - 1/2x^2$

And the circle $x^2 + (y + 3/4)^2 = (5/4)^2$

WA 4 Plot 1/2 - .5x^2, ((25/16) - x^2)^.5 – .75 from x = -1 to 1

We see the trajectories are closer now.

WA 5 Arc Length ((25/16) - x^2)^.5 – .75,from x = -1 to 1
Ans. 2.318

WA 6 Arc Length .5 - .5x^2, from x = -1 to 1
Ans. 2.296

It appears to me that the lower we toss the ball the closer the fit would be, but that is another question.

What we do see is that Aristotle was not too far off in this case. Nevertheless, he was wrong.

The truth is we should always be suspect of any math model of some physical phenomenon.

No math model is a totally accurate model. There are always simplifying assumptions. For example, Galileo would be wrong here if we did not ignore air resistance, or the rotation of the Earth.

The more powerful our tools, the better models we can construct.

Here is an example, where Galileo thought he had another good model, and then when a more powerful tool, calculus, came along it was shown there was an even better model.

A Hanging Chain --- Catenary

What shape does a chain of uniform density form when it is hung from two parallel points?

Galileo thought it was a parabola. Unlike the trajectory, he had no way to prove it. After the calculus was invented, several mathematicians were able to prove that the shape was what is called a catenary.

The shape of a catenary is the graph of cosh(x)

$= (e^{x} + e^{-x})/2$.

What is the difference between the catenary and a parabola?

The parabola through the points (0,b) and (a,0) and (-a,0) is $y = (a - b)x^2 + b$

cosh(0) = 1, and cosh(1) = cosh(-1) = 1.543

So, $y = .543x^2 + 1$ is the parabola passing through the catenary at (-1, cosh(-1)),(0,cosh(0)), and (1,cosh(1))

WA 7 Plot cosh(x), .543x^2 +1 from x = -1 to 1

We can see why Galileo thought it was a parabola

WA 8 Arc length cosh(x) from x = -1 to 1

Ans. 2.3504

WA 9 Arc length .543x^2 +1 from x = -1 to 1

Ans. 2.3427

Wow. They are really close. Easy to see why Galileo thought this was a parabola

Let's see just how much area is between these two curves

WA 10 Integrate cosh(x)- .543x^2 -1 from x = -1 to 1

Wow. Only .0116 difference in area.

There are a several lessons to be learned.

No matter how close a fit a math model seems to have it may not be perfect, and there may be a better one.

With a tool like Wolfram Alpha (WA) it is very easy to explore different models. This exercise would have been much more time consuming and difficult with the classical techniques and tools of calculus.

Just, how great were our ancestors?

Aristotle may be the 'father' of modern logical reasoning, we talk about Aristotelian logic.

Galileo may have been the 'father' of modern science.

Both, made some mistakes given the tools of their time.

Liebniz and others corrected Galileo on the catenary, but they had much more powerful tools, i.e. calculus.

Newton created a math model for physics which is used to this day. However, it was not as accurate as Einstein's model, just as Galileo's model was not as accurate as Liebniz's for the catenary.

Indeed, we couldn't have such things as GPS with Newton's model.

Newton's model for physics was replaced by Einstein's.

Einstein's models don't work for the nano-world. There we need Quantum Theory.

And, on and on.

Each model gets replaced by a more accurate model as the tools of math are developed.

It behooves any STEM student to keep up to date on the latest tools.

One place to begin is to be sure you learn calculus and differential equations using a tool like Wolfram Alpha.,

The calculus program, Tier 5, in Triad Math's Ten Tier high school math program utilizes WA, and is revolutionarily different that a standard calculus course.

The ε – δ 'theory' arguments are replaced by modern infinitesimal arguments, making the reasoning more intuitive and heuristic.

Techniques of Integration are now effectively obsolete and replaced by the tool Wolfram Alpha.

The result is a student can learn the concepts of calculus and how to solve any calculus problem that arises in STEM much quicker and easier than in a classical course.

More importantly, now the student can solve problems that are not even attempted in a classical calculus course, but necessary for many STEM subjects.

Wolfram Alpha does to the classical tools of calculus what the scientific calculator did to the slide rule and log and trig tables for pre-calculus.